U.S. Department of Housing and Urban Development
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The FHA Home Equity Conversion Mortgage Insurance Demonstration

A Model to Calculate Borrower Payments and Insurance Risk

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I. Introduction

This paper is a technical explanation of the payments model developed for the FHA Home Equity Conversion Mortgage (HECM) insurance demonstration. Being primarily a technical document, this paper presents the model in sufficient detail to allow actuaries, economists, and other specialists to replicate and evaluate the payments allowed under the demonstration. Due to the relative novelty of home equity conversion mortgages (also known as "reverse" mortgages) in the marketplace, this paper is also written to convey the basic underlying concepts to executives and generalists within the finance community. The confidence of the latter groups in the soundness of its foundation is important to the success of the HECM demonstration.

The fundamental reasons that reverse mortgages to date have not enjoyed wider market acceptance as vehicles for home equity conversion by elderly households are due to the inherent risks in these loans and not necessarily to any lack of demand for an efficiently designed and priced instrument.[1] Reverse mortgages present risks to both lenders and borrowers. The borrower's primary risk is that of outliving his or her assets (home equity being the primary asset for many potential borrowers), while that of the lender is the risk of earning less than the market rate of return on the investment. The ways in which these risks are managed has a direct bearing on the pricing of the mortgage and its ultimate marketability.

For an example of the interaction between risks, pricing, and marketability, consider an individual lender operating in a regional market. Without mortgage insurance, this lender is unkikely to be able to generate the volume or geographic dispersion necessary to minimize diversifiable risks in a reverse mortgage portfolio.[2] If the uninsured

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[1] Determination of whether demand exists for reverse mortgages is one of the objectives of the HECM demonstration. Thus, a final determination as to the existence of a significant level of demand is premature at this time. We do note, however, that the market potential is large (the 1987 American Housing Survey estimates that there are 3.2 million elderly households with incomes below $15,000 who own their homes) as is the volume of initial inquiries received by HUD's toll-free HECM information phone line (over 28,000 inquiries received in 1989).

[2] The lender's risk of earning less than market rate can be separated into two categories: non-diversifiable (or fundamental) risk, and diversifiable risk. An example of non-diversifiable risk is national economic recession which can cause property values to fall in all regional markets, thereby resulting in a larger than anticipated number of loan defaults (in the case of reverse mortgages, loan balances which exceed property values). Such risks cannot be reduced by geographic diversification, which is the process of distributing loans among many regional markets, because the nationwide recession affects all markets. In technical terms, the probabilities of loss due to national recession are

lender offered a reverse mortgage with all the design features of the HECM demonstration, he would have to charge a substantial premium to cover the risks involved. The added cost of this risk premium would surely reduce demand for that lender's product. Alternatively, the same lender could have designed a less risky (from his own perspective) reverse mortgage product, such as one which placed strict limits on the size of cash advances and which required repayment of the loan at the end of the term. With less risk to begin with, this modified product could be offered at a greatly reduced cost. From the borrower's perspective, however, the modified product offers much less protection from the possibility of outliving one's assets. Thus the lower price may not improve the marketability.

The FHA HECM demonstration was designed to manage risks more efficiently. It will allow lenders to offer elderly borrowers a relatively risky product (i.e. one which offers substantial borrower protection) without charging a large risk premium. The pricing advantage is achieved by insuring lenders against losses arising from both diversifiable and non-diversifiable risks. Since the demonstration is not a subsidy program, the FHA must collect a premium to offset insurance losses. But because FHA can achieve both high volume and geographic diversification, the risks are minimized by pooling. The resulting premium is lower than that which a regional, uninsured lender could offer. The demonstration will show whether the trade-off between risks and pricing has produced a marketable product.

The design process which resulted in the FHA HECM payments model grew out of a recognition of the trade-off between risks and pricing. After consulting widely with representatives of the finance community (both primary and secondary mortgage markets), advocates of the elderly, and government agencies at the federal, state, and local level, the HUD Office of Policy Development and Research developed the HECM concept using a three-step process:

1. Determine the levels of protection that borrowers and investors (lenders) would require,

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interdependent. An example of diversifiable risk is regional economic recession in which one market area experiences falling property values due to declines in the industries which predominate that region's economic base. Geographic diversification reduces the risk in this case because not all regions are affected. That is, the probabilities of loss due to regional resession are independent. Another diversifiable risk is individual borrower longevity (causing the reverse mortgage balance to exceed property value without a recession). Diversification to reduce risk in this case requires a large volume of mortgages rather than geographic diversification. With a large volume, the independent probabilities of borrower longevity make losses predictable through application of the law of large numbers.

2. Fix a uniform premium structure for the mortgage insurance, and

3. Control the risk of each mortgage by producing a factor table which limits cash advances (payments) to borrowers.

This paper focuses on the payments model which implements the third step in the above process. The remaining text is organized as follows. Setion II describes the basic payments model which produced the factor table. Section III details the relationship between the factors and limits on cash advances to borrowers. A final section deals with the key actuarial assumptions and the sensitivity of the model to changes in these assumptions. The appendix contains the mathematical derivation of the model's key equations.

II. The Basic Payments Model

II-A. Introduction

This section presents the underlying assumptions and the mathematical equations of the HECM payments model. At the most basic level, the HECM demonstration provides mortgage insurance to private lenders (or in some cases, quasi-public lenders such as state housing finance agencies) who make cash advances secured by a "reverse" mortgage and note to qualified elderly borrowers. The loans need not be repaid until the borrower moves, sells the property, or dies (although the borrower may prepay at any time). The debt is non-recourse, which means that if the borrower is unable to repay the loan when due, the lender looks only to the value of the mortgaged property for repayment and not to any other assets of the borrower or the borrower's estate.

In cases in which the loan balance has grown to exceed the value of the property (either because the borrower lived in the property a long time, during which interest and other loan costs kept accruing, or because the property may have declined in value) the lender would not be repaid the full principal and interest due. By not receiving the full principal and interest, the lender would be earning less than the market rate of return on such a loan. This is precicely where the HECM insurance comes in. The insurance pays the lender the shortfall (within certain limits which are not relevant to the discussion of the basic model) between the property value and the principal and interest owed at the time the loan becomes due and payable. As the economists would say, the insurance gives the lender a "put" option to sell the mortgage and note to the insurer at a "strike price" equal to the outstanding balance.

Due to the non-recourse nature of the debt, the value of the insurance (put option) is closely related to the future levels of the debt and property value. Assuming a fixed interest rate on the note and a specified pattern of cash advances to the borrower, future debt levels are predictable.[3] Future property values are not. Even if the long run annual average property appreciation rate is assumed constant, variations in individual property appreciation rates will result in significant differences from the average. This is particularly true the farther into the future the estimates are to be made. Thus, the key component of the HECM payments model is its specification of the random, or stochastic, nature of future property values.

The remainder of Section II is organized into the following subsections. The first is an explanation of the stochastic process used to predict future property values. Next comes a discussion of mortgage insurance claims on both traditional mortgages and reverse mortgages. Then we introduce the fundamental relationship of the payments model, which requires the value of the insurance (i.e. the investor's put option) to be less than or equal to premium revenue, and which achieves equality, or break-even, only at maximum utilization. The next two subsections detail the actuarial implementation of the fundamental relationship: one calculates the expected mortgage insurance premium, the other prices the option by computing expected losses. A final subsection defines maximum utilization as the principal limit factor, and considers the excess premium collection if utilization is less than the maximum.

II-B. Predicting Future Property Values

The first fundamental assumption of the model is the specification of future property values. The assumption is that these values can be simulated by a stochastic geometric Brownian motion process. This process, based on probability theory, is often referred to as a log-normal random walk.[4] According to Ross (1983), geometric Brownian motion is useful in

[3] After the basic model has been developed, the assumptions of fixed interest and specified cash advances will be relaxed.

[4] To justify the specification of future property value as a geometric Brownian motion process, consider the mortgaged property as a zero-coupon "real" asset. This assumes the return on owner-occupied housing excludes imputed rent (a reasonable assumption because the mortgage insurer has no interest in imputed rent) and includes only the expected price inflation, μ, plus a stochastic term, σ, to describe deviations from expected inflation. The resulting differential equation ($dH/H = \mu dt + \sigma dz$) describes a geometric Brownian motion process if the inflation parameters, μ and σ, are constant. (Note that dH and dt are differentials of house value and time, respectively, and dz is the differential of a stochastic variable which is normally distributed with mean 0 and standard deviation 1). See Malliaris and Brock (1982), Chapter 4, for a discussion of stochastic inflation processes. Furthermore, see Cunningham and Hendershott (1984), or Epperson, Kau, Keenan, and Muller (1984) for their explicit use of the geometric Brownian motion process to model housing assets.

The geometric Brownian motion process can also be described by the following equation:

modeling when the percentage changes (and not the absolute changes) are assumed to be independent and identically distributed[5].

$$H(t) = H_0 e^{Y(t)},$$

where

$H(t)$ is the property value at some future time, t,

H_0 is the initial property value,

$Y(t) = \mu t + \Phi(t)$,

μt is the expected inflation, or drift term, and

$\Phi(t)$ is a standard Brownian motion, or Weiner process, with $E[\Phi(t)] = 0$ and $\mathrm{var}[\Phi(t)] = \sigma^2 t$.

Note that because a standard Brownian motion $\Phi(t)$ is normally distributed for all $t > 0$, the distribution of $H(t)$ is log-normal for all $t > 0$. For more details of the mathematical properties of stochastic processes see Ross (1983).

[5] Some would argue that annual percent rates of change in house price are not independent; rather, that they are serially correlated. Serial correlation means that the change in any given year is at least in part determined by the changes which occurred in previous years. Example: do two consecutive years of appreciation above (below) the mean imply that a third year of appreciation above (below) the mean is more likely than not to occur? The random walk model says no, while the serial correlation model says yes. Case and Schiller (1989) examine the efficiency of the real estate market, looking to find evidence of independent price changes (efficient market, random walk model applies), or serially correlated price changes. Although their study finds evidence of correlation in city-wide price indices over short time periods, it nevertheless is inconclusive with regard to accepting or rejecting the random walk model for predicting individual house price changes (the volatility of individual house prices was found to be of far greater magnitude than the serial correlation found in the index).

In a separate study, Case (1986) examined housing prices over time in a single market (Boston), concluding that prices in the mid 1980's clearly grew above levels that market "fundamentals" (i.e. population and employment growth, cost of construction, etc.) could explain. He offers the explanation of a temporary price "bubble", which occurs when expectations of future appreciation drives prices up (or down) without corresponding shifts in market fundamentals (for example, speculation). This results in serial correlation, at least in the short run. Such price bubbles are not believed to be sustainable over time. Gau (1987) also notes the existence of temporary price bubbles in real estate markets, but goes on to conclude that studies have not been able to reject the hypothesis that real estate markets are efficient. If so, the random walk

An implication of the random walk assumption is that the annual appreciation rate of each property is treated as an independent observation of a normally distributed random variable with constant mean, μ, and standard deviation, σ. As will be discussed further in Section IV, the mean, or average, appreciation, μ, is assumed to be 0.04 (i.e., 4 percent annual rate), and the standard deviation of appreciation, σ, is assumed to be 0.10 (i.e. 10 percent annual rate). The geometric Brownian motion (or log-normal random walk) process is also referred to as a "diffusion" process because the cumulative appreciation rates of each property over time are also normally distributed, but with growing mean, μt, and standard deviation, $\sigma\sqrt{t}$ (where t is elapsed time expressed in years). As Fig. 1 illustrates, each property begins at a point value ($H_0 = 1.0$) which is the initial appraised value. Future point values of the property are unknown due to the random appreciation rates; however, the value distribution of a pool of properties is known due to the assumed distribution of the appreciation rates. Specifically, the growing mean and standard deviation of the cumulative appreciation rate causes the future value distribution to widen, or diffuse, as t increases. Fig. I illustrates the diffusion by showing the locus of points on the house value axis for which cumulative appreciation rates are one standard deviation from the mean. (According to the properties of a normal distribution, 68 percent of the observed values should fall within one standard deviation of the mean.)

II-C. Mortgage Insurance Claims

FHA mortgage insurance on traditional, or "forward", mortgages is similar to insurance on reverse mortgages in that both give the investor a put option to sell the loan to the FHA at par. The option may not be exercised while the mortgagor is in full compliance with the note, or loan agreement. That is, as long as the forward mortgage borrower continues to make monthly payments, keeps the property in good repair, and meets all other terms and conditions of his or her mortgage note, the investor has no valid claim to insurance benefits. The same applies to reverse mortgages: as long as the borrower remains in occupancy, keeps the property in good repair, and meets all terms and conditions of the note, the investor has no claim for mortgage insurance.[6] This is an important point. It is only

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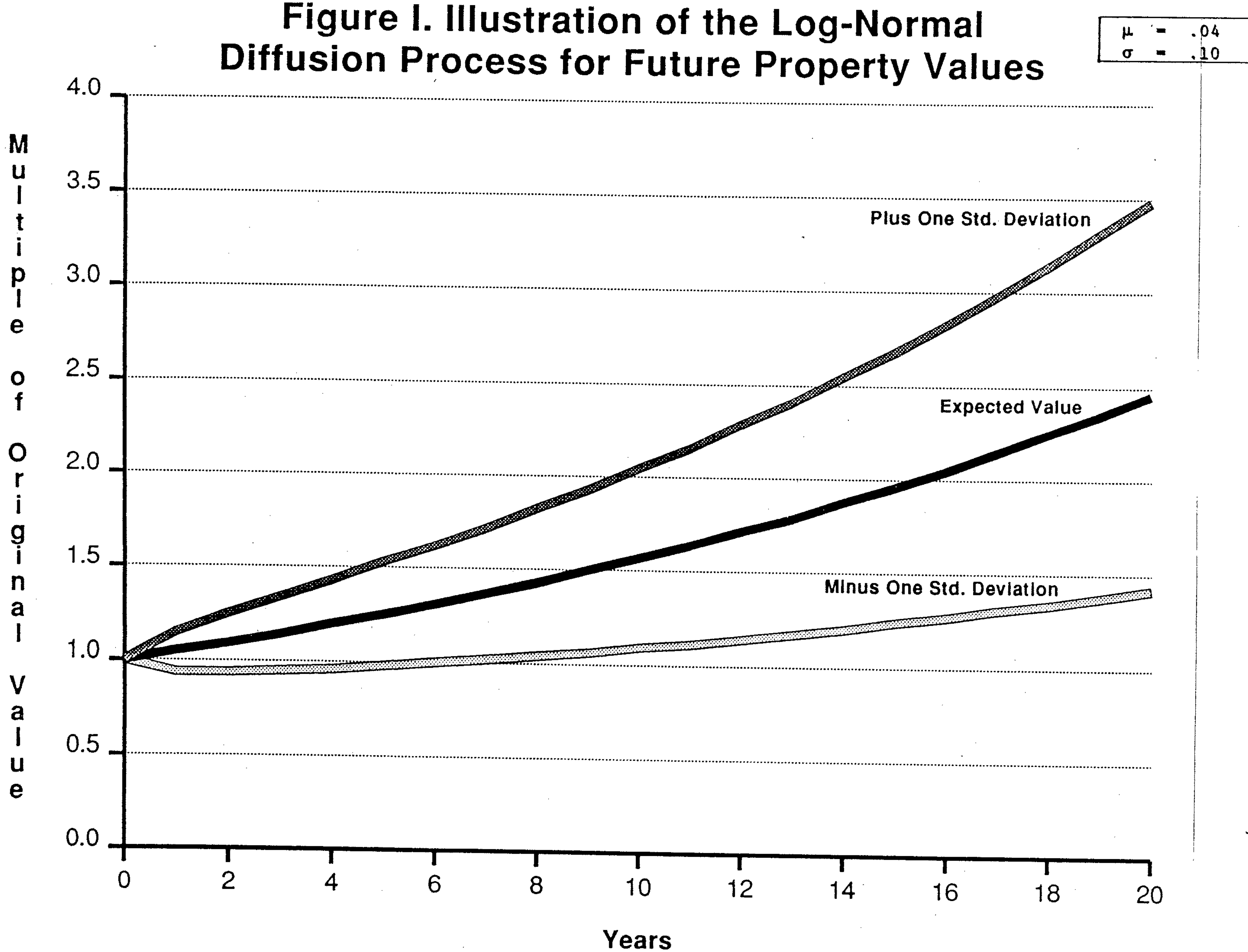
model may still be appropriate, particularly in the long run. (A long run perspective is appropriate for the HECM program because reverse mortgage insurance losses are "back loaded" in comparison to traditional, or "forward" mortagages, for which losses are "front loaded". HUD is currently undertaking a study of house price changes to fill some of the many gaps which exist in our knowledge of the subject.)

[6] One exception is when the originating lender choses the "assignment option" for receiving HECM insurance benefits. Under this option, the investor of record at the time that the loan balance grows to equal the maximum claim amount has a brief window during which he may assign the loan and receive insurance benefits even though the borrower is in full compliance with the note agreement. The existence of the assignment option does not affect the discussion of the basic payments model.

Figure I. Illustration of the Log-Normal Diffusion Process for Future Property Values



through some action of the borrower, such as a default for failure to make monthly payments in the case of a forward mortgage, that gives the investor the right to exercise the option. In the case of reverse mortgages, the borrower is not obligated to make monthly payments, so examples of borrower actions which trigger the investor's option to file for insurance benefits are death, or a move out of the mortgaged property (perhaps into a nursing home). The differences in the triggering of insurance claims on forward vs. reverse mortgages may seem very minor, but there is one significant difference. There is a tendency for forward mortgages to default systematically during times when the borrower equity is negative (caused by a drop in property value).[7] This is not likely to occur with reverse mortgages. Terminations of reverse mortgages are expected to be actuarially predictable and not sensitive to equity levels.[8]

Differences that mortgage insurers are likely to experience due to the above are as follows. Traditional mortgage insurers experience a surge in claim rates during an economic down cycle; reverse mortgage insurers should find claims more evenly distributed between economic ups and downs.[9] The property values of forward mortgages which terminate (both claim and non-claim terminations) are not usually representative of the value

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[7] For a review of the economics literature pertaining to forward mortgage default, see Neal (1989).

[8] Under the option model of default, a borrower will exercise his option whenever the gains to default are greater than the costs. For traditional mortgages this is when the following inequality holds:

$$MV - H + R > C,$$

where $(MV - H)$ is the recapture of negative equity (MV is the market value of the mortgage, and H is the current value of the property), R is the free rent gained between default and foreclosure, and C represents transaction costs associated with default. See Foster and Van Order (1985). For reverse mortgages, the equation must be modified as follows:

$$(B - A) - H + R > C,$$

where $(B - A)$ is now the market value of the mortgage (B is the outstanding balance and A is the present value of future cash advances). The recapture of negative equity $(B - A - H)$ is never greater than zero due to the non-recourse nature of the FHA reverse mortgage note (unlike forward mortgages where the borrower may owe more than the property is worth). The free rent, R, is zero because the note already allows the borrower to remain in occupancy as long as he chooses. The left hand side of the equation never exceeds the right hand side; hence, there is no financial incentive for borrowers to terminate the mortgage during periods of negative equity.

[9] There is likely to be some increase in reverse mortgage claims in such circumstances because more borrowers will find themselves in a negative equity position. However, without the systematic defaults brought on by negative equity, reverse mortgage claims should not increase nearly as much as forward mortgage claims.

distribution of the pool. Instead, properties with below average appreciation will be over-represented among forward mortgage terminations, and the properties with loans remaining in force will no longer have the log-normal value distribution. The latter will have a distribution resembling the log-normal but with a greatly reduced low value tail due to defaults. The property values of reverse mortgages which terminate (both claim and non-claim terminations) will be much more representative of the value distribution of the pool. A second fundamental assumption of the model is therefore that properties with loans remaining in force will keep the log-normal distribution.[10]

Preservation of the log-normal distribution for loans remaining in force will make the task of pricing reverse mortgage insurance easier as will be shown below. A mathematical value estimate of the reverse mortgage insurance option can be derived from the properties of the log-normal price distribution. Traditional mortgage insurance, on the other hand, must be valued by another method. Such methods include regression models of past claim experience, or option models which rely on simulation rather than mathematical techniques. The regression method is not possible for reverse mortgages due to the lack of past claim experience, and the simulation method, while possible, would be cumbersome.

II-D. The Fundamental Relationship

The fundamental relationship of the basic payments model requires the present value of the mortgage insurance on a pool of mortgages to be less than or equal to the present value of the premium collected. If at each future point in time we can estimate the expected loss due to payment of insurance claims and the expected premium based on a declining number of mortgages remaining in force, then the fundamental relationship can be expressed as follows:

$$\sum_{t=0}^{\infty} \{ E[L(t)]\ (1+i)^{-t} \} \leq \sum_{t=0}^{\infty} \{ E[MIP(t)]\ (1+i)^{-t} \}, \qquad (1)$$

where

$E[\cdot]$ is the expected value operator,

$L(t)$ is the loss incurred in period t,

i is the periodic discount rate,

$MIP(t)$ is the mortgage insurance premium collected in period t, and,

[10] Some systematic terminations of reverse mortgages may occur among those borrowers whose equity increases due to unusually high appreciation. These borrowers may refinance their reverse mortgages or convert the increasesd equity to cash by selling the property. To the extent that this occurs, the policies in force may deviate from the log-normal distribution.

t is an arbitrary unit of time, which we define as months. Note that the summations from month zero to infinity reflect the fact that HECM mortgages have no stated maturity.

The loss function, L(t), and the premium function, MIP(t), from expression (1) are actually functions of several variables. The borrower's initial age, the initial property value, the interest rate charged on cash advances, and the amounts and timing of cash advances all affect these estimates. They are shown as functions of time only in expression (1), because in the basic model, we assume an age, initial property value, interest rate, and pattern of cash advances before applying the fundamental relationship. Once the specification of the basic model is complete, the effects of changing all these variables will be discussed (Section III).

Note that expression (1) is not an equation, but rather an inequality. Equality between the two summation terms is achieved only when the borrower receives the maximum cash advances to which he or she is entitled. For example, consider a borrower of given age, with a given property value, who gets a reverse mortgage with a set interest rate. If this borrower wanted to receive a single cash advance from the lender on the first day of the mortgage, forgoing any future cash advances from the lender, what is the maximum amount he or she could receive? The answer is the largest amount for which expression (1) still holds. At the maximum, the expression will be an equation. Therefore, by replacing the inequality sign in (1) with an equal sign, we can solve for the desired maximum cash advance. To accomplish this, we need to develop the equations that will allow us to evaluate both sides of the fundamental relationship.

II-E. Expected Mortgage Insurance Premium

Recall from earlier discussion of the three-step HECM design process that step two involved adoption of a uniform premium structure. This premium structure is 2 percent of the initial property value paid up-front, plus 1/2 percent (50 basis points) annually (but billed monthly) on the growing loan balance.[11] The up-front portion is paid at the time of loan settlement or closing; hence, it is certain to be collected. The 50 basis point annual premium, on the other hand, is to be paid in the future, and will be collected as scheduled only if the loan remains in force. For a pool of mortgages, varying amounts of premium will be collected depending on how long each mortgage remains in force. The expected mortgage insurance premium, representing an estimate of the average premium collection over all mortgages in the pool, can be calculated using a

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[11] The HECM program places a limit on the amount of initial property value that can be considered when determining maximum payments to borrowers. This limit is used to determine a quantity defined as the "maximum claim amount", which is the lesser of the initial property value or the program limit for the locality. Therefore, the up-front portion of the premium will actually be 2 percent of the maximum claim amount. For purposes of specifying the basic model, this distinction is not relevant.

prepayment function that gives loan survival probabilities over time. Such a function is described below.

Let l_{x+t} be the probability that a HECM loan originated by a borrower with initial age x is still in force t months after origination. If we could specify these probabilities, then the expected premium in month t would be given by equation (2):

$$E[MIP(t)] = l_{x+t}\, MIP(t)\ , \qquad (2)$$

where

MIP(t) is the scheduled MIP in month t, and

l_{x+t} is as defined above with additional properties discussed below.

Note that $l_x = 1$ because all loans are in force at the time of origination ($t = 0$). If T is the month in which the borrower will turn 100 years old, then by assumption we have $l_{x+T} = 0$. The latter assumes that all loans will be terminated by the time the borrower attains age 100, placing an actuarial limit on the summation terms in expression (1). The intermediate values l_{x+t} (for $0 < t < T$) are probabilities between 1 and 0. They resemble an actuarial life table for borrowers between the ages of x and x+T. The probabilities are in fact smaller than the borrowers' actuarial survival probabilities to account for loan prepayments and terminations for reasons other than death of the borrower.

The model uses the U.S. Decennial Life Tables for 1979-81 (DHHS Publication No. [PHS] 85-1150-1) to compute the l_{x+t}. These tables are sometimes referred to as the "general population" life tables. The female table is used because female life expectancy is longer than that of males, the majority of HECM applicants is expected to be single females, and we believed that the establishment of separate HECM factor tables for males and females (with corresponding differences in maximum cash advances by gender) would be subject to court challenge. There are many issues involved with the choice of a life table upon which to base the loan survival probabilities. These issues will be dealt with in more detail in Section IV. The remainder of this section will only discuss the numerical techniques involved in transforming the above referenced table into the required probabilities to implement equation (2).

The female general population life table must be adjusted and interpolated to produce the required monthly probabilities. The adjustment for loan terminations for reasons other than death of the borrower is referred to as the "move-out factor". The move-out factor used in the model is based on data from the general elderly population. These data show that elderly homeowners in the general population have move-out rates that decline with advancing age when expressed as a percentage of the age specific death rate (see Jacobs (1988)). The explanation is that the younger elderly population more frequently makes voluntary moves into more suitable housing for their retirement years, often converting some of their equity through sale of the home. The older elderly population is less mobile, generally moving only when a greater level of health care services than can be provided in the home is required. Based on data from several sources, Jacobs estimated these rates to range from 519% of mortality for the 65-69 year old group down to 47% of mortality for the 85+ group. Since the HECM program provides an incentive to remain in the home, move-out factors equal to the experience of the general population would be inappropriate. Instead of a declining move-out rate as a percentage of the age specific death rate, the HECM payments model used a constant rate of 30 percent for all ages. This rate is less than that of the 85+ group in the general population, and as such was believed to be sufficiently conservative.

A second adjustment to be made is to convert the U. S. Decennial Life Table data into survival probabilities. The published table used is entitled "number living at beginning of age interval," and is defined as follows. For each age in whole years, the table starts with a hypothetical cohort of 100,000 live female births, and shows the number who will survive to attain the indicated age. For example, the table shows that 67,186 females (out of the original cohort of 100,000) will survive to age 75 or greater. Similarly, the survivors to age 76 are estimated to be 64,910. For a 75 year old HECM borrower, the probability of attaining age 75 is 1 (he or she has already attained that age). To compute all future annual survival probabilities for the 75 year old, we must divide the table entries for all subsequent years by 67,186. Thus the probability of this 75 year old surviving to attain age 76 is 0.9661 (64,910 ÷ 67,186). For a 65 year old HECM borrower, the probability of surviving to age 76 is only 0.7772 (64,910 ÷ 83,520, where 83,520 represents the number of survivors at age 65).

The model computes annual survival probabilities, $S_{i,j}$, as described above for i = {62, 63, ..., 99}, and j = {i, i+1, ... ,100}. Note that i is the initial age in years, while j is the attained age in years. Note also that $S_{i,i} \equiv 1$, and $S_{i,101} \equiv 0$ for all i. To convert these annual borrower survival probabilities into monthly loan survival probabilities, l_{x+t}, we use the following equation which both interpolates geometrically and adjusts for move-outs:

$$l_{x+t} = (S_{i,j}\,[S_{i,j+1}/S_{i,j}]^{r/12})^{1+m}\,, \qquad (3)$$

where

i = initial age in years = {62,63, ... , 99},
j = attained age in full years = {i, i+1, ... , 100},
x = initial age in months = 12i,
t = attained age minus initial age in months = 12(j-i)+r,
r = months between attained ages j and j+1 = {0, 1, ... , 11}, and
m = move-out rate expressed as a decimal = 0.3.

Using the earlier example of the 75 year old borrower, we compute the initial loan survival probability as:

$$l_x = (1.0\ [0.9661 \div 1.0]^{0/12})^{1.3} = 1.0\ ,$$

where i = 75, j = 75, and r = 0. The loan survival probabilities estimated for the next two months are:

$$l_{x+1} = (1.0\ [0.9661 \div 1.0]^{1/12})^{1.3} = 0.9963\ , \text{ and}$$
$$l_{x+2} = (1.0\ [0.9661 \div 1.0]^{2/12})^{1.3} = 0.9926\ ,$$

where i and j are again 75, but r = 1 and 2, respectively.

II-F. Expected Losses

Perhaps the key component of the HECM model is the technique used to evaluate the investors' option contained in the mortgage insurance contract. The value of this option for a given initial age of the borrower, initial property value, interest rate on cash advances, and pattern of cash advances depends on the loan survival probabilities, l_{x+t}, as described above, and on the mathematical properties of the stochastic process assumed for predicting future house prices. Since investors are likely to exercise their option whenever optimal to do so, its value is the present value of expected insurance claim losses. A loss occurs on those loans which terminate at a time when the outstanding balance exceeds the house value.

Let B(t) represent the outstanding balance on the mortgage at time t. If the mortgage is terminated during time t (either due to death of the borrower or other reason), and if the value of the house is greater than B(t), then there is no loss. If, on the other hand, the mortgage is terminated at time t, and the house value is less than B(t), there is a loss. The magnitude of the loss, if one occurs, is denoted L(t). It is estimated to be the difference between the amount owed, B(t), and the conditional expected value of the house given that the value is less than B(t).

Since only a small percentage of the loans in a pool will incur a loss during a particular month, we need to compute the expected loss for each month into the future. The expected loss at time t, therefore, is an average loss for all loans in the pool. It is denoted E[L(t)]. It is computed as the amount of loss, L(t), from above times the liklihood of such a loss occurring at time t. The latter is estimated to be the probability of termination multiplied by the probability that the house value is less than the balance, both evaluated at time t. A method of calculating all of these amounts is presented below.

The probability of loan termination at a given time is analogous to the calculation of the probability of death from an actuarial survival table. Let d_t denote the probability of loan termination during month t.

Then, from the properties of the loan survival probabilities, l_{x+t}, as previously defined, we have:

$$d_t = l_{x+t} - l_{x+t+1}. \qquad (4)$$

To compute the probability that the house value is less than the balance, $B(t)$, we first define $b(t) = B(t)/H_o$, where H_o is the original property value. Recall from the discussion of the geometric Brownian motion process that the cumulative appreciation rate (i.e., the natural log of $H(t)/H_o$) is normally distributed with mean μt and standard deviation $\sigma\sqrt{t}$. Utilizing the probability density function of the normal distribution (see equation A-2 in Appendix 1), we express the desired probability as:[12]

$$A(t) = [1/\{\sigma\sqrt{t}\ \sqrt{(2\pi)}\}] \int_{-\infty}^{\ln[b(t)]} e^{-\frac{1}{2}[(y-\mu t)/\sigma\sqrt{t}]^2}\, dy. \qquad (5)$$

Equation (5) represents the area under the normal probability density function to the left of the value $\ln[b(t)]$. The model uses numerical approximation methods (Simpson's rule[13] for example) to evaluate the improper integral.

In order to express the conditional expected value of the house given that the value is less than $B(t)$, we first find an expression for the unconditional expected value of the house at time t. The mean, or unconditional expected value, of the house is derived in Appendix 1 and is given by:

$$E[H(t)] = H_o\, e^{\mu t + \frac{1}{2}\sigma^2 t}. \qquad (6)$$

Intuitively, equation (6) represents the mean of the entire log-normal house price distribution at time t. The conditional expected value, on the other hand, is similar in concept, except that it represents the mean of only the left tail of the log-normal distribution, up to the value $B(t)$. Thus, the conditional expected value is always less than the unconditional expected value. In fact it is calculated by multiplying the unconditional expected value from equation (6) by a factor β. This factor ranges in value from zero to one (i.e., $0 < \beta < 1$), and is calculated by numerical evaluation of a second improper integral:

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[12] Note that the constants μ and σ were previously defined as annual rates ($\mu = .04$ and $\sigma = .10$), but in expression (1) we defined the unit of time to be months. Hence, the values of μ and σ appearing in equations (5) through (8) must be converted to monthly equivalents: $\mu = .04 \div 12 = .003333$, and $\sigma = .10 \div \sqrt{(12)} = .02887$.

[13] See Press, Flannery, Teukolsky, and Vetterling (1986), Chapter 4.

$$\beta = [1/A(t)]\,[1/\sqrt{(2\pi)}] \int_{-\infty}^{U(t)} e^{-\frac{1}{2}(y-\sigma\sqrt{t})^2}\, dy\ , \tag{7}$$

where

$U(t) = \{\ln[b(t)] - \mu t\}/\sigma\sqrt{t}$, and
$A(t)$ is given in equation (5).

Equation (7) is derived in Appendix 1. Using this result, we now have an expression for the conditional expected value of the house, H(t), given that the value is less than B(t):

$$E[H(t)|h < B(t)] = E[H(t)]\ \beta = H_0\ e^{\mu t + \frac{1}{2}\sigma^2 t}\ \beta\ . \tag{8}$$

Using equations (4) through (8) we can express the expected loss at time t succinctly as the product of three terms:

$$E[L(t)] = \{B(t) - E[H(t)|h < B(t)]\}\ [d_t]\ [A(t)]. \tag{9}$$

II-G. Principal Limit Factor

Equations (1) through (9) specify the basic HECM payments model. If given the initial age of the borrower, the initial property value, a fixed rate of interest accrual on cash advances, and an assumed pattern of cash advances, an insurer can use the basic model to underwrite the reverse mortgage loan. The insurer chooses the stochastic parameters to use in the model (μ and σ), as well as the survival probabilities ($S_{i,j}$), the move out factor (m), and the discount rate (i). If analysis of a loan application indicates that the fundamental relationship is valid (i.e. expression (1) holds), then the loan is acceptable for insurance. If not, then the present value of cash advances must be reduced (either by nominal reduction in payment amounts, or by real reduction--i.e. deferral of payments into the future). Limiting the present value of cash advances is the way the insurance risk is managed in the model because borrower age, property value, and interest rate are not generally within the control of the insurer. Risks could have been managed using a risk-adjusted premium structure, but this option was rejected in favor of the fixed premium structure (less confusion among lenders used to fixed premiums under FHA programs).[14]

Although the basic model may be used to analyze any pattern of cash advances to the borrower, one particular pattern is worthy of detailed analysis. This is the case in which the borrower receives the maximum cash advance in a lump sum on the first day of the mortgage, and receives no

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[14] Theoretically, some of the the risk could be managed by using the underwriting process to change the variables that are controlled by the insurer (for example, medical information similar to that used by life insurance underwriters could be gathered to modify the $S_{i,j}$ on a case-by-case basis). But such procedures are rejected for many reasons, including their doubtful legality under the HECM demonstration.

further cash from the lender at any time in the future. One advantage of analyzing this pattern of cash advances is that its present value is equal to its nominal value. Management of risk through changes in the present value of cash advances is greatly simplified in this case. Excessive risk can be made acceptable by a simple reduction in the nominal value of an up-front lump sum cash advance. If expression (1) does not hold, then reduce the amount of the advance until it does.

The maximum up-front cash advance is the amount for which strict equality exists in expression (1). For any given borrower age and interest rate, the ratio of this amount to the initial property value remains constant for all property values.[15] This is the **principal limit factor** for the given age and interest rate combination. Multiply the principal limit factor by the initial property value and the result is the maximum up front cash advance the borrower could receive at that rate of interest.

To illustrate, Table I shows that .416 is the principal limit factor for a 75 year old borrower when the interest rate is 10 percent. Using a house value of $100,000 merely as a round number (any value would work), the table lists annual cash flows of an individual mortgage for 25 years (at which time the borrower would be 100 years old, and by assumption, the loan could not extend beyond that time). The maximum up-front cash advance of $41,600 (.416 x 100,000) consists of the $2,000 initial mortgage insurance premuim paid on behalf of the borrower plus the net amount of $39,600 cash received by the borrower. (Note that the borrower could have received $41,600 in cash if the MIP were paid out of pocket. In either case, the initial balance on the loan is limited to $41,600). The table then lists end of year calculations of equations (3), (5), (6), and (8). Finally, it shows the annual expected MIP and expected losses for a pool of identical mortgages, along with the present values of these amounts (discount rate = 9.5%). The bottom line is that the sum of the present values of the expected MIP and the expected losses both equal approximately $4230.

If the borrower in the above example had opted to receive less than the $41,600 up-front, then the bottom line comparison of expected premium and losses would show an excess of premium collected. Specifically, had the borrower taken only $31,200 (75 percent of the maximum), the present value of expected premium would be $3674, but the present value of expected losses would be only $1510, an excess of $2164. Conversely, had the borrower been given an amount above the $41,600, there would be a shortfall in the premium collection. The insurer will accept the former loans for which borrowers take less than the maximum, but they will refuse to insure the latter. Excess premium collection may result if the insurer's book of business includes loans below the maximum determined by the principal limit

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[15] This is true in the basic model which assumed no limitation on the value of the property for determining payments. In the HECM program, there is a limit on the property value. Therefore, the ratio will be constant for property values less than or equal to the limit.

TABLE 1

Home Equity Conversion Mortgage Insurance Demonstration - Payments Model Illustration
Verification of Principal Limit Factor

Loan Terms:				
Borrower Age:	75	Closing Costs:	1500 (Included in Cash Advance)
Interest Rate:	10.0	Servicing Fee:	0
House Value:	100000		
Maximum Claim Amount:	100000	Principal Limit Factor:	0.416

	Individual Mortgage Cash Flows					End of Year Calculations				Mortgage Pool Averages			
										Expected MIP		Expected Loss	
year	Begin Balance	Cash Advances	Interest Accrued	MIP Paid	End Balance	House Exp Val	Probility Bal > Val	Condit Exp Val	l(x+t)	Nominal	Pres Val	Nominal	Pres Val
0	-	39600	-	2000	41600	100000	0.0000	-	1.0000	2000	2000	0	0
1	41600	0	4366	218	46184	104603	0.0000	-	0.9562	214	205	0	0
2	46184	0	4848	242	51274	109417	0.0000	-	0.9104	227	197	0	0
3	51274	0	5382	269	56925	114454	0.0000	54757	0.8625	239	189	0	0
4	56925	0	5974	299	63198	119722	0.0010	59876	0.8122	251	181	0	0
5	63198	0	6633	332	70163	125232	0.0066	65392	0.7595	261	171	1	0
6	70163	0	7364	368	77895	130996	0.0228	71323	0.7045	270	161	4	2
7	77895	0	8175	409	86479	137026	0.0540	77685	0.6476	277	150	16	8
8	86479	0	9076	454	96009	143333	0.1011	84494	0.5895	281	139	43	21
9	96009	0	10077	504	106590	149930	0.1618	91763	0.5310	283	127	97	44
10	106590	0	11187	559	118336	156831	0.2319	99503	0.4730	282	115	187	76
11	118336	0	12420	621	131377	164050	0.3072	107726	0.4160	277	103	318	118
12	131377	0	13790	689	145856	171601	0.3836	116439	0.3606	269	91	496	167
13	145856	0	15308	765	161929	179499	0.4580	125650	0.3080	257	79	712	219
14	161929	0	17040	805	179774	187761	0.5283	135364	0.2592	242	68	953	266
15	179774	0	18869	943	199586	196403	0.5930	145587	0.2148	224	57	1205	306
16	199586	0	20948	1047	221581	205443	0.6514	156321	0.1748	205	47	1459	338
17	221581	0	23256	1163	246000	214899	0.7033	167569	0.1393	183	39	1700	358
18	246000	0	25819	1291	273110	224791	0.7489	179334	0.1085	160	31	1893	363
19	273110	0	28665	1433	303208	235137	0.7885	191620	0.0826	137	24	2012	351
20	303208	0	31823	1591	336622	245960	0.8226	204430	0.0615	115	18	2047	325
21	336622	0	35330	1767	373719	257281	0.8517	217769	0.0446	94	14	2001	289
22	373719	0	39224	1961	414904	269123	0.8764	231644	0.0317	75	10	1876	247
23	414904	0	43547	2177	460628	281511	0.8973	246063	0.0219	58	7	1689	202
24	460628	0	48346	2417	511391	294468	0.9149	261037	0.0149	45	5	1467	160
25	511391	0	53672	2684	567747	308022	0.9296	276578	0.0000	33	3	3908	373

Present Value Expected Total Premium: 4231
Present Value Expected Total Losses: 4233

factor. Section III will address excess premium collection in more detail.

A complication arises from the fact that not every borrower is likely to request cash advances up-front in a lump sum. Some may want level monthly payments for a specified term (term mortgage). Others may want level payments for as long as they occupy the home (tenure mortgage). Others may not want monthly payments at all, opting instead for a line of credit. Finally, some may want to combine reduced monthly payments with a small up-front lump sum or line of credit. The basic model could be used to analyze the "bottom line" premium and loss expectations for virtually any pattern of cash advances. Unfortunately, there are too many possible cash flow patterns to have a maximum utilization factor (such as the principal limit factor discussed above) for each. As the next section will show, there is a way to use the principal limit factor to simplify the analysis of different cash advance patterns. The technique is an approximation, meaning that there will be some minor differences between the limits determined from the principal limit and those that the basic model would suggest. But the deviation from the fundamental relationship in expression (1) will be small, and the simplicity and flexibility gained will be significant.

III. Determining Cash Advance Payments

III-A. Transition from Basic Model to HECM Payments Model

According to Hogg and Klugman (1984), the premium that an insurer must charge for insurance is the sum of the following three components:

1. Pure premium,

2. Expenses of doing business, and

3. Risk charge.

Simple definitions of the above are as follows. The pure premium is the average amount of loss due to payment of claims. The expenses of doing business are items such as salaries, employee benefits, rental of office space and equipment, etc. Finally, the risk charge is the payment that the insurer requires for exposing the surplus in its loss reserve fund (capital investment) to risk fluctuations. All three are expressed as costs per unit of risk exposure. The transition from a basic insurance model as developed in the preceding section of this paper to the HECM payments model, which implements the insurance demonstration, involves an assessment that the premium to be collected will cover all three components. Each is adressed below.

The basic insurance model is essentially the calculation of the pure premium. Expression (1) in the basic model assures us that the premium to be collected will cover expected losses under the model's assumptions. If actuarial experience deviates from assumptions, average losses due to payment of claims could be underestimated. The risk charge is used to maintain a capital reserve sufficient to cover these losses. If the model is specified properly, the size of deviations from the expectations will be relatively small. In such cases, the appropriate risk charge will be relatively small too. If the model is poorly specified, the deviations from expectations will be greater, resulting in a commensurately greater risk charge.

Recall from Section II of this paper, the least predictable of the actuarial variables needed to evaluate expression (1) is the future property value. Note that the model could have specified future property value as a non-stochastic function of time.[16] However, such a specification would be a poor one, because property values must be predicted many years into the future in the model. Instead, the stochastic

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[16] For example, the equation expressing future house prices could have been:

$$H(t) = H_o e^{\mu t} ,$$

where $H(t)$ is the property value at time t, H_o is the initial property value, and μ is a constant annual rate of appreciation.

specification allows future property values to vary considerably about their expected values without violating the model's actuarial assumptions. In this way the model manages the insurance risk more effectively. It computes expected losses more accurately than a non-stochastic model would, and thereby places less reliance on the contingency reserve to cover unexpected losses.

The second component of the premium, the expense of doing business, was not introduced explicitly into the basic model. Neither is it explicit in the premium structure of the HECM program. The reason is that the current premium structure is set up to collect more than the pure premium. This will be explained in more detail in Section III-F below which discusses the premium reserve.

If administrative expenses were made explicit, they would amount to about 10 basis points annually on the loan balance[17]. At this level, they would amount to about $400 (present value) on a reverse mortgage to a 75 year old borrower with a $100,000 house in a 10 percent interest rate environment. If charged explicitly, they would reduce cash advances by about 5 percent. Instead, a policy decision was made to leave cash advances as calculated, with expenses of doing business paid out of the premium reserve.

Finally, the third premium component, the risk charge, compensates the insurer in the event the parameters of the basic model were chosen in error. It does this by building and maintaining a capital reserve to cover unexpected losses. For example, on a given book of business (say policies endorsed in a given year), the long run property appreciation rate might average only 2 percent instead of 4 percent. Or the move-out rate might turn out to be significantly lower than 130 percent of the general population mortality rate. In such cases the insurance losses would be greater than predicted on average for that book, causing a reduction in the capital reserve. Of course, the errors in parameter estimation on another book of business could easily work in the other direction. That is, 6 percent appreciation could be realized, as could move-outs at a rate above 130 percent of population mortality. Insurance losses on the latter book would be smaller than predicted. The latter would raise the capital reserve. Over time, the level of capital fluctuates, and according to Borch (1990) it behaves like a random walk. If program experience over time shows that capital is being depleted (i.e., the fluctuations are biased downward), then the assumptions may have to be reassessed, and the insurance repriced. Absent such program experience, the parameters should be conservatively chosen to compute reasonable expected losses. The actuarial assumptions will be discussed in more detail in Section IV of this paper, along with a sensitivity analysis to changes in the parameter values.

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[17] This estimate is comparable to the annual rate that FHA currently allows for these expenses in the Section 203(b) mortgage insurance program. Absent actual program experience, an estimate specific to the HECM program is not available.

The HECM program being a demonstration, the model assumes that it begins with sufficient capital. Furthermore, since premiums are more frontloaded than are losses, the cash position of the fund will start out positive regardless of the level of capitalization. The risk premium for maintaining the necessary capital is not explicit in the HECM demonstration. It is also assumed to be covered by the premium reserve, which is discussed in Section III-F.

The remainder of Section III is devoted to the relationship between the basic model and the calculation of cash advances to borrowers using the HECM payments model. The text is organized into the following subsections. The first two are definitional--one being a detailed explanation of the principal limit, the other defining the expected average mortgage rate and the discount rate. The next subsection gives the equations for calculating term and tenure cash advance payments from the principal limit--i.e., "the principal limit method" of the payments model. Next comes a comparison of the cash flow patterns produced by the principal limit method with the basic model, showing that the method provides a good approximation to the basic model. The final subsection estimates excess premium collection from likely cash flow paterns.

III-B. Principal Limit Defined

Recall from Section II that the principal limit factor for a given borrower age and interest rate was defined in the basic model as the ratio of the maximum up-front cash advance divided by the initial property value. The HECM payments model uses the same factors. It does not adjust them because the expenses of doing business and the risks not covered by the basic model are assumed to be covered by the excess premium reserve. The only change is a definitional one. Specifically, the principal limit factor in the payments model is expressed as a ratio of the maximum up-front cash advance divided by the maximum claim amount, where the latter is defined as the lesser of the initial property value or the maximum mortgage amount that FHA can insure for a one-family dwelling in the area (i.e., the Section 203(b) mortgage limit).

The principal limit is not the same as the principal limit factor. The principal limit is an increasing function of time. Its initial value is equal to the principal limit factor (for a given borrower age and interest rate) times the maximum claim amount. The initial principal limit is, therefore, the maximum up-front cash advance, given the borrower's age, the interest rate on the loan, and the initial property value. The future values of the principal limit represent the outstanding balance on the loan assuming the borrower received the maximum up-front cash advance. Together, the initial principal limit and its future values represent the maximum loan balance at any point in time when the borrower takes an up-front cash advance. As we shall see, the key concept in the "principal limit method" is the definition of the **principal limit as the**

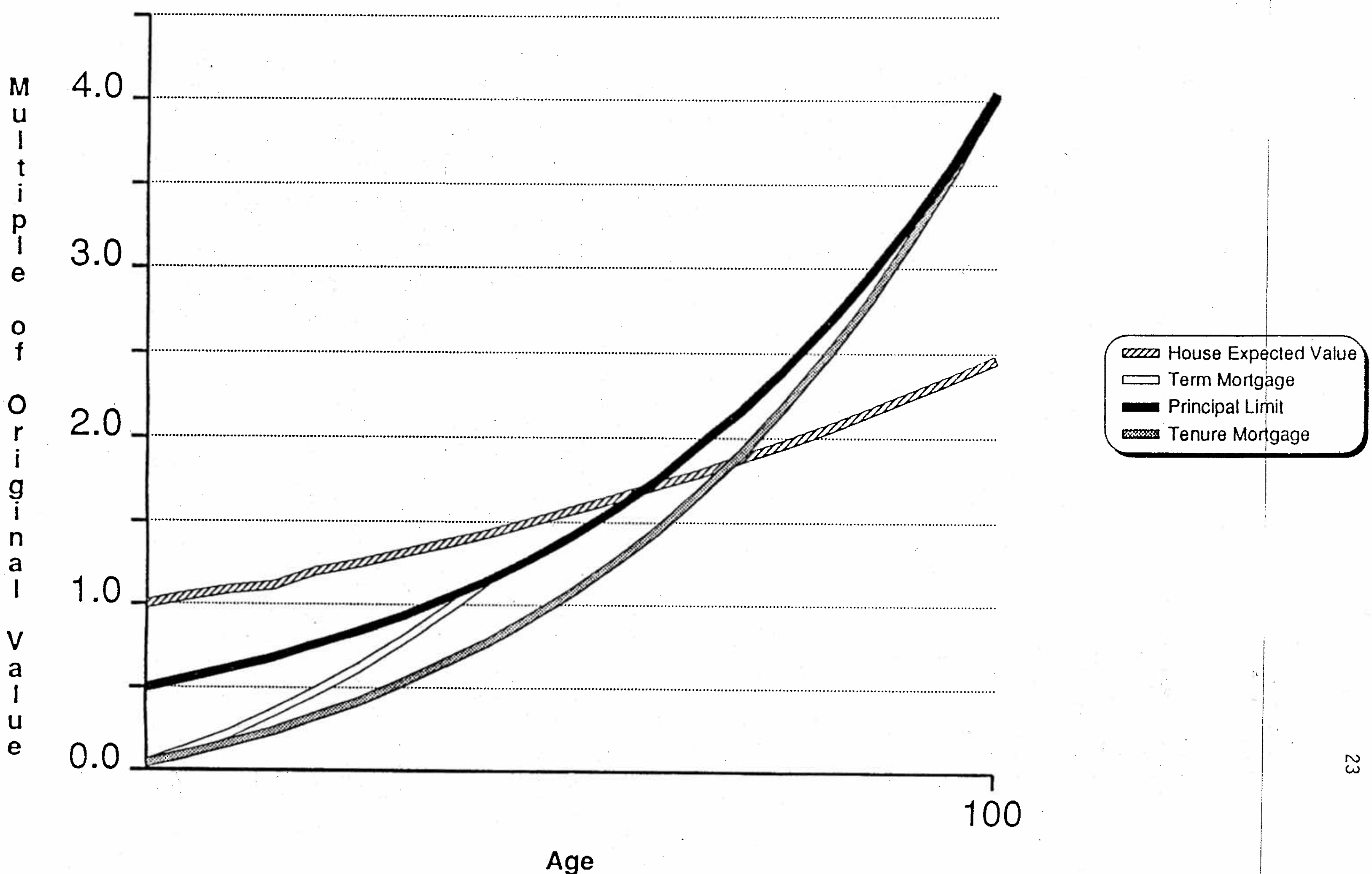
upper bound of the loan balance at any point in time for all patterns of cash advances.

Figure 2 illustrates the relationships between the principal limit, the expected value of the property, and loan balances for term and tenure cash advance patterns. At origination, the principal limit is less than the initial property value. In fact, it is calculated as the principal limit factor times the maximum claim amount (maximum claim amount is assumed to equal the property value in Fig.2). The initial principal limit is the maximum up-front cash advance. If the borrower receives this amount up-front, forgoing any future cash advances from the lender, then the loan balance would follow the principal limit curve over time. The principal limit curve, thus defined, is also the maximum loan balance for term, tenure, or line-of-credit cash advance patterns. If the borrower lives long enough, the principal limit (and the maximum loan balance) eventually grows to exceed the expected property value.

If the borrower elected to receive cash advances in equal monthly installments under a term or tenure payment plan, then the initial loan balance would be much lower than either the initial property value or the principal limit. The initial loan balance on term and tenure payment plans equals the loan closing costs. Notice how the loan balances for term and tenure mortgages in the early years of the loan grow at a faster rate (i.e., have a greater slope) than the principal limit curve. This is because the principal limit curve is growing only by accrued interest and mortgage insurance premium. The term and tenure balances are growing by accrued interest, mortgage insurance premium, and by the monthly cash advances to the borrower. Note also that the term loan balance grows to equal the principal limit at the end of the term. When this occurs, no further cash advances are available to the borrower, and future balances on the term loan follow the principal limit curve. The tenure loan balance, on the other hand, remains below the principal limit throughout the life of the loan because monthly cash advances do not stop. The tenure loan balance converges with the principal limit only when the borrower's age is 100. Thus the tenure mortgage is similar to a term mortgage in which the term was calculated to expire at age 100.

If the borrower elected a line-of-credit mortgage rather than up-front cash or regularly scheduled monthly cash advances, the borrower may request cash advances in any amount desired and at any time subject to the principal limit. That is, whenever the loan balance, including cash advanced plus accrued interest and MIP charges, reaches the principal limit, the line-of-credit is exhausted. At this point the borrower may not receive any further cash, but may continue to reside in the property. Once the line-of-credit is exhausted, the loan balance follows the principal limit curve. Note that the cash advance pattern in which the borrower receives the maximum up-front amount is actually a special case of the line-of-credit mortgage. In this case the borrower exhausts the line-of-credit on the first day by withdrawing the full amount of the initial principal limit.

Fig. 2. House Expected Value, Principal Limit, Term and Tenure Mortgage Balances



III-C. Interest Rates

Up to this point, the definitions of principal limit factor and principal limit have assumed that the interest rate on the loan was fixed. The HECM program allows the loan rate to be adjustable, and it is expected that many loans will take advantage of this feature. The reason is that a fixed rate reverse mortgage exposes the lender (investor) to a considerable amount of interest rate risk by requiring funds to be advanced to borrowers in the future at today's fixed lending rate. The cost of the lender's interest rate risk will be passed along to borrowers. Thus many borrowers are likely to opt for less costly adjustable rate loans.

Adjustable rate HECM loans will have initial rates equal to a specified interest rate index plus a fixed margin. The index is the one-year Treasury rate[18], while the margin is set by agreement between the borrower and the lender at the time of closing. Future adjustments to the loan rates will be determined by the index rate at the time of the adjustment, plus the margin, with interest rate adjustment caps and ceilings as prescribed in the loan agreement.[19]

How does the HECM payments model deal with adjustable rate loans when the basic model requires a fixed interest rate to calculate the principal limit factors? To answer this question, we borrow from a model which describes the term structure of interest rates. Richard (1978) shows that under an assumption called the "expectations hypothesis", the equilibrium yield[20] on a long term default-free bond (such as a ten-year Treasury security) equals the average of expected future short term yields over the same time period. That is, under the expectations hypothesis, the ten-year Treasury yield is an estimate of the average combined yields of the current one-year Treasury plus nine expected future one-year Treasury yields.[21]

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[18] The index actually used is the most recent weekly average yield for U. S. Treasury bonds and notes, adjusted to a constant maturity of one year, as determined by the U. S. Treasury and published by the Board of Governors of the Federal Reserve. References in the text to the ten-year Treasury rate are similarly defined except that the weekly average yields are adjusted to a constant maturity of ten years.

[19] For HECM's which provide for annual adjustments of the loan rate, the annual adjustment cap is 2 percent, and the life of the loan ceiling is 5 percent. For HECM's which provide for monthly adjustments of the loan rate, there is no monthly adjustment cap, and the life of the loan ceiling is determined by agreement between the borrower and the lender.

[20] Equilibrium in this context refers to standard arguments in which bonds and derived securities are priced to avoid arbitrage opportunities.

[21] To be precise, two points must be clarified regarding the expectations hypothesis. The first is that the equilibrium condition actually applies to default-free discount bonds as opposed to coupon bearing Treasuries due to the distortions caused by taxation. This distortion is minimized by averaging the yields of various coupon Treasury notes and bonds having a specified average maturity. The second is that the equilibrium yield for such bonds (of any maturity) equals the expected average of all future spot

An alternative to the expectations hypothesis is the "liquidity preference theory." This alternative states that due to the uncertainty of future interest rates and to the generally risk-averse nature of investors, the yield on longer term securities must contain a liquidity, or risk, premium to induce investors to hold them instead of shorter term securities. The two theories are not mutually exclusive. When combined with the expectations hypothesis, the liquidity preference theory implies that the ten-year Treasury yield is greater than the average combined yields of the current and future one-year Treasury yields by an amount equal to the liquidity premium.[22]

The HECM model makes use of the above theories to define a fixed interest rate proxy called the **expected average mortgage rate**, or **expected rate**, for all loans. For fixed rate loans, the expected rate is simply equal to the fixed rate. For adjustable rate loans, the expected rate is an estimation of the average loan rate over the first ten years of the loan. Since the rate to be averaged is the one-year Treasury rate plus a fixed margin (subject to caps and ceilings as previously indicated), the expected rate is defined as the ten-year Treasury rate at the time of closing plus the same margin. This proxy represents the market's best estimate of the average adjustable loan rate over the specified period.

The expected rate is the fixed interest rate used to calculate the principal limit factor. It is used in the basic model to predict future loan balances, as well as to determine the discount rate (see below). On fixed rate HECM loans, this represents no change. On adjustable rate loans, the expected rate allows the principal limit factor to be estimated without complicating the model with variations in interest rates. The factor determined with the expected rate is generally lower than if the initial adjustable rate were entered into the model as a fixed rate. The reason is that the expected rate is usually higher than the initial loan rate (just as the ten-year Treasury rate usually exceeds the one-year Treasury rate, reflecting market expectations and liquidity preferences). A higher rate in the basic model makes loan balances accrue faster,

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rates during the remaining term of the bond. (The current spot rate is the instantaneous risk-free interest rate. Future spot rates are unknown, but investors make assumptions about them which are reflected in their pricing of bonds of various maturity. The latter gives rise to the Treasury yield curve. See Richard (1978)). Thus the ten-year Treasury yield equals the average of current and future one-year Treasury yields, as derived from the yield curve and investor expectations of future spot rates.

[22] According to Richard (1978), there are some economists who believe that the liquidity premium can be negative in certain circumstances, particularly when future consumption is valued more highly than present consumption. For purposes of the HECM model, the assumption is that there is more likely to be a positive liquidity premium than a negative one.

resulting in higher expected losses, and therefore, a lower principal limit factor.[23]

In addition to determining the principal limit factor, the expected rate is also used to compute future values of the principal limit, as illustrated in Fig. 2. Even though loan balances will accrue interest at the variable loan rate, all future values of the principal limit will be determined using the fixed expected rate. This means that on adjustable rate loans, the actual loan balances may eventually exceed the principal limit curve if loan rates exceed the expected rate. Furthermore, the limits on monthly term or tenure payments, and cash advances on line-of-credit mortgages will be derived from the principal limit and the expected rate. This will be discussed in detail in the next sub-section.

One final definition must be given before the equations of the "principal limit method" can be introduced. This is the discount rate used in the basic model. **The discount rate, like the expected rate, is a fixed rate that does not change over the life of the loan.** It is defined to be the expected rate minus one half percent (50 basis points). For fixed rate loans this is equivalent to the loan rate minus one-half percent. Since reverse mortgage fixed rates are likely to be set by lenders at some margin above the ten-year Treasury rate, the discount rate for all HECM's can be considered to equal the 10-year Treasury rate plus a margin minus one-half percent. Depending on the size of the margin, the discount rate will generally be a little greater than the 10-year Treasury rate.

III-D. Equations of the Principal Limit Method

We are now ready to discuss the equations that govern maximum cash advances to borrowers under the HECM payments model. The model incorporates equations (1) through (9) of the basic model, which produced the principal limit factors. However, the HECM payments model includes three additional equations, given below, to derive the maximum cash advances for all types of HECM loans.

We begin with a formal definition of the principal limit. The principal limit at any time t is given by:

$$\mathbf{PL(t) = F(x,R)\ MCA\ (1+c)^{t},} \tag{10}$$

where

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[23] Note that the definition of expected rate does not attempt to remove the liquidity premium from the 10-year Treasury rate. The inclusion of the premium, which is generally believed to be a positive amount, raises the expected rate, thereby lowering the principal limit factor. The liquidity premium becomes, in effect, a risk premium which protects the insurer from interest rate fluctuations that differ from expectations.

PL(t) is the principal limit at time t,

F(x,R) is the principal limit factor for a borrower of age x and fixed interest rate, R, which equals the expected rate,

MCA is the maximum claim amount, which is lesser of the property value or the FHA maximum insurable mortgage for the area,

c is the periodic compounding rate which equals the expected rate, R, plus the 1/2 percent annual mortgage insurance premium charge converted to a monthly rate, and

t is the number of months after loan origination ($0 \leq t \leq T$), with T being the value of t for which the borrower turns 100 years old.

Note that at origination (t=0), the principal limit is equal to the principal limit factor times the maximum claim amount as we would expect. Table 2 illustrates the calculation of principal limit at origination and and at selected time periods after origination.

Next we define the **net principal limit** at time t:

$$\mathbf{NPL(t) \;=\; max[\;0,\; PL(t) - B(\underline{r},t)],} \tag{11}$$

where

NPL(t) is the net principal limit at time t,

$\max[q_1,q_2]$ is maximum of the quantities, q_1 and q_2.

PL(t) is the principal limit given by equation (10), and

$B(\underline{r},t)$ is the actual loan balance at time t, including cash advances, interest accrued (using the actual loan rates given by the vector $\underline{r} = (r_0, r_1, \ldots, r_t)$ where r_j is the loan rate in effect during month j), and MIP.

Note that at origination (t=0), the net principal limit equals the initial principal limit minus the initial cash advance on the loan[24]. Note, too, that the maximum function prevents the net principal limit from being negative in cases in which the balance grows to exceed the principal limit due to actual rates, r_j, exceeding the expected rate, R.

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[24] The HECM demonstration also accounts for set-asides from the principal limit which further reduce the net principal limit. The right hand side of equation (11) should actually be written as the greater of zero or the quantity given by: $PL(t) - B(\underline{r},t) - S(t)$, where S(t) is the set-aside amount at time t. Such set-asides are those for repairs, first-year taxes, and those for future loan servicing fees not included in the interest rate. See HUD Handbook 4235.1 for more detail.

TABLE 2

Estimated Principal Limits at Origination and at Selected Time Periods after Origination*

(Expected) Interest Rate: 10 %
Appraised Value = $ 100,000

		- - Months after Origination - -		
Age	At Origination	60	90	120
62	$ 24,700	41,659	54,102	70,262
65	28,000	47,225	61,331	79,650
70	34,200	57,682	74,911	97,286
75	41,600	70,163	91,120	118,337
80	50,000	84,330	109,519	142,231
85	58,900	99,341	129,013	167,549

* Assumes:

Appraised value is within FHA max. mortgage amount for area.

Interest rate on loan may be fixed or adjustable.

TABLE 3

Estimated Maximum Monthly Cash Advance Payments for Selected Term Mortgages Compared With Tenure Mortgages *

(Expected) Interest Rate: 10 %
Appraised Value = $ 100,000

	Term Mortgages (months)			
Age	60	90	120	Tenure Mortgage
62	$ 452	338	284	187
65	522	391	328	218
70	654	490	411	278
75	812	608	510	357
80	991	742	622	460
85	1180	884	741	607

* Assumes:

Initial payment to borrower covers closing costs and fees of $3500, with no additional cash advanced at closing.

Appraised value is within FHA max. mortgage amount for area.

Term mortgage payments stop at term expiration, but debt repayment is deferred if borrower remains in occupancy.

Loan servicing charges are included in the loan interest rate, which may be a fixed or adjustable rate.

For borrowers who wish a line-of-credit, NPL(t) given by equation (11) is the maximum cash advance available at each time t. Once the net principal limit goes to zero, the line-of-credit is exhausted.

For borrowers who wish level monthly cash advances, either for a specified term, or for as long as the borrower remains in occupancy of the property (tenure), one additional equation is required. The maximum monthly advance is given by:

$$p(t,m) = [NPL(t)\ (1+c)^{m}]\ [c \ / \ \{(1+c)^{m+1} - (1+c)\}\ ], \qquad (12)$$

where

p(t,m) is the monthly cash advance beginning in month t+1 and continuing for a term of m months,

NPL(t) is the net principal limit at time t from equation (11),

c is the monthly compounding rate given by the expected rate, R, plus the 1/2 percent annual MIP charge converted to a monthly rate,

$m = (T - t)$ for tenure, where T is the value of t for which the borrower turns age 100, and

$0 < m < (T - t)$ for term, with m chosen by the borrower.

Note that the right hand side of (12) consists of the product of two algebraic expressions contained within brackets. The first is the future value of the net principal limit projected ahead to the end of the term (i.e., for m months). The second is the formula for a standard sinking fund monthly contribution that will grow to $1 at the end of the term. The rate of interest in both cases is the compounding rate defined above. Their product gives the monthly cash advance which with compound interest will grow to equal the future value of the net principal limit at the end of the term.

Note also that in the special case where t = 0, equation (12) gives the maximum monthly cash advance beginning in the first month of the loan. For all other values of t, the same equation allows monthly advances to be modified by the borrower at some future date. Table 3 illustrates tenure and term cash advance payments calculated at origination.

For borrowers who wish to combine reduced monthly cash advances with a small line-of-credit, the same three equations may be used. In such a case the borrower is allowed to split the principal limit in equation (10) into two accounts, one designated for the line-of-credit, the other for monthly cash advances. The loan servicer must maintain separate balances for each account, but otherwise, equations (10) through (12) remain valid. If the borrower with separate principal limit accounts wishes to modify the payment plan at some future date, the net principal limit in either account may be switched to the other account. For example, a borrower who originally established a small line-of-credit along with tenure payments, may decide in the 60^{th} month to switch the unused net principal limit from the line-of-credit account into the tenure payment account, and use equation (12) to calculate a higher monthly tenure payment for the remainder of the mortgage.

III-E. Advantages of the Principal Limit Method

There are three advantages to using the principal limit method given by equations (10) through (12) rather than using the basic model to determine maximum cash advances. These are: simplicity of implementation, equity among payment options, and flexibility in restructuring. The simplicity is due to the fact that a single factor corresponding to each borrower age and interest rate combination can be used to determine maximum cash advances regardless of the pattern desired by the borrower. The equity argument is based on the fact that borrowers are free to choose the pattern of cash advances which best suits their needs, knowing that any pattern will have the same present value[25]. The flexibility is a corrollary of the equity argument: a borrower is allowed to restructure future cash advances at any time as long as the revised cash advances have the same present value.

Tables 4 and 5 use the basic model to analyze term and tenure maximum monthly cash advances calculated by the principal limit method. The format of the tables is identical to that of Table 1. The monthly cash advances are taken from Table 3: a 75 year old borrower can receive \$510 monthly under a ten-year term plan (\$6116 annually), or \$357 monthly under a tenure plan (\$4279 annually). As we noted previously, the basic model can be used to analyze any pattern of cash advances. Thus Tables 4 and 5 show individual mortgage cash flows generated by the principal limit method, while showing expected losses and MIP calculated by the basic model. The sum of the present values of expected losses and premium are no longer equal, as they were in Table 1. The explanation is that maximum payments calculated using the principal limit method are only approximations of the maximum payments that would be generated by the basic model[26]. The tables show that the principal limit method results in undercollection of premium on term mortgages, and excess premium collection on tenure mortgages. The

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[25] This is true if the borrower uses the compounding rate defined in equation 10 as the discount rate. In practice, borrowers use different discount rates, depending on whether current or future consumption is more highly valued. Thus, some borrowers will prefer cash advances to be concentrated in the early years, while others will prefer cash advances deferred into the future.

[26] More specifically, both the expected MIP and expected loss calculations in the basic model are functions of the loan balance at any point in time. Thus neither is independent of the pattern of cash advances. For example, a tenure mortgage has lower loan balances at every point in time than a mortgage in which the borrower received the maximum lump sum at closing. Thus the tenure mortgage will produce less premium and smaller losses.

TABLE 4

Home Equity Conversion Mortgage Insurance Demonstration - Payments Model Illustration
Analysis of Ten Year Term Mortgage

Loan Terms:				
Borrower Age:	75	Closing Costs:	1500 (Included in Cash Advance)
Interest Rate:	10.0	Servicing Fee:	0
House Value:	100000		
Maximum Claim Amount:	100000	Principal Limit Factor:	0.416

	------ Individual Mortgage Cash Flows -----					---- End of Year Calculations -----				------ Mortgage Pool Averages -----			
										--Expected MIP--		--Expected Loss--	
year	Begin Balance	Cash Advances	Interest Accrued	MIP Paid	End Balance	House Exp Val	Probility Bal > Val	Condit Exp Val	l(x+t)	Nominal	Pres Val	Nominal	Pres Val
0	-	1500	-	2000	3500	100000	0.0000	-	1.0000	2000	2000	0	0
1	3500	6116	710	35	10361	104603	0.0000	-	0.9562	35	33	0	0
2	10361	6116	1429	71	17977	109417	0.0000	-	0.9104	67	58	0	0
3	17977	6116	2229	111	26433	114454	0.0000	-	0.8625	99	78	0	0
4	26433	6116	3116	156	35821	119722	0.0000	-	0.8122	131	94	0	0
5	35821	6116	4102	205	46244	125232	0.0000	44161	0.7595	161	106	0	0
6	46244	6116	5195	260	57815	130996	0.0006	54235	0.7045	190	114	0	0
7	57815	6116	6409	321	70661	137026	0.0089	64881	0.6476	217	118	1	0
8	70661	6116	7758	388	84923	143333	0.0437	76028	0.5895	240	119	9	5
9	84923	6116	9255	463	100757	149930	0.1200	87597	0.5310	260	117	48	21
10	100757	6116	10917	546	118336	156831	0.2319	99503	0.4730	275	112	155	63
11	118336	0	12420	621	131377	164050	0.3072	107725	0.4160	277	103	318	118
12	131377	0	13789	689	145855	171601	0.3836	116438	0.3606	269	91	496	167
13	145856	0	15308	765	161929	179499	0.4580	125649	0.3080	257	79	712	219
14	161929	0	16995	850	179774	187761	0.5283	135364	0.2592	242	68	953	266
15	179774	0	18868	943	199585	196403	0.5930	145586	0.2148	224	57	1205	306
16	199585	0	20948	1047	221580	205443	0.6514	156320	0.1748	205	47	1459	338
17	221580	0	23256	1163	245999	214899	0.7033	167568	0.1393	183	39	1700	358
18	245999	0	25819	1291	273109	224791	0.7489	179334	0.1085	160	31	1893	363
19	273109	0	28664	1433	303206	235137	0.7885	191620	0.0826	137	24	2012	351
20	303206	0	31824	1591	336621	245960	0.8226	204430	0.0615	115	18	2047	325
21	336621	0	35329	1767	373717	257281	0.8517	217769	0.0446	94	14	2001	289
22	373717	0	39224	1961	414902	269123	0.8764	231644	0.0317	75	10	1876	247
23	414902	0	43547	2177	460626	281511	0.8973	246063	0.0219	58	7	1689	202
24	460626	0	48345	2417	511388	294468	0.9149	261036	0.0149	45	5	1467	160
25	511388	0	53673	2684	567745	308022	0.9296	276577	0.0000	33	3	3908	373

Present Value Expected Total Premium: 3545
Present Value Expected Total Losses: 4171 (Losses will be lower if move-outs increase after year 10)

TABLE 5

Home Equity Conversion Mortgage Insurance Demonstration - Payments Model Illustration
Analysis of Tenure Mortgage

Loan Terms:				
Borrower Age:	75	Closing Costs:	1500 (Included in Cash Advance)
Interest Rate:	10.0	Servicing Fee:	0
House Value:	100000		
Maximum Claim Amount:	100000	Principal Limit Factor:	0.416

	------ Individual Mortgage Cash Flows -----					---- End of Year Calculations -----				------ Mortgage Pool Averages ------			
										--Expected MIP--		--Expected Loss--	
year	Begin Balance	Cash Advances	Interest Accrued	MIP Paid	End Balance	House Exp Val	Probility Bal > Val	Condit Exp Val	l(x+t)	Nominal	Pres Val	Nominal	Pres Val
0	-	1500	-	2000	3500	100000	0.0000	-	1.0000	2000	2000	0	0
1	3500	4279	607	30	8416	104603	0.0000	-	0.9562	30	28	0	0
2	8416	4279	1124	56	13875	109417	0.0000	-	0.9104	52	45	0	0
3	13875	4279	1695	85	19934	114454	0.0000	-	0.8625	75	59	0	0
4	19934	4279	2332	117	26662	119722	0.0000	-	0.8122	98	70	0	0
5	26662	4279	3038	152	34131	125232	0.0000	-	0.7595	119	78	0	0
6	34131	4279	3822	191	42423	130996	0.0000	-	0.7045	140	83	0	0
7	42423	4279	4692	235	51629	137026	0.0002	48438	0.6476	159	86	0	0
8	51629	4279	5658	283	61849	143333	0.0023	57054	0.5895	175	87	0	0
9	61849	4279	6730	337	73195	149930	0.0125	66237	0.5310	189	85	2	1
10	73195	4279	7923	396	85793	156831	0.0401	75975	0.4730	199	81	11	5
11	85793	4279	9244	462	99778	164050	0.0912	86253	0.4160	206	77	41	15
12	99778	4279	10711	536	115304	171601	0.1649	97052	0.3606	208	71	107	36
13	115304	4279	12342	617	132542	179499	0.2544	108348	0.3080	207	64	225	69
14	132542	4279	14150	708	151679	187761	0.3508	120118	0.2592	201	56	399	111
15	151679	4279	16159	808	172925	196403	0.4463	132339	0.2148	192	49	622	158
16	172925	4279	18390	919	196513	205443	0.5354	144987	0.1748	180	42	883	204
17	196513	4279	20865	1043	222700	214899	0.6151	158046	0.1393	164	35	1161	244
18	222700	4279	23613	1181	251773	224791	0.6841	171499	0.1085	147	28	1419	272
19	251773	4279	26665	1333	284050	235137	0.7426	185337	0.0826	128	22	1621	283
20	284050	4279	30052	1503	319884	245960	0.7914	199556	0.0615	109	17	1746	277
21	319884	4279	33813	1691	359667	257281	0.8315	214156	0.0446	90	13	1785	258
22	359667	4279	37989	1899	403834	269123	0.8643	229144	0.0317	73	10	1734	228
23	403834	4279	42624	2131	452868	281511	0.8909	244531	0.0219	57	7	1607	192
24	452868	4279	47770	2389	507306	294468	0.9123	260335	0.0149	44	5	1427	155
25	507306	4279	53485	2674	567744	308022	0.9296	276577	0.0000	33	3	3897	372

Present Value Expected Total Premium: 3201
Present Value Expected Total Losses: 2880

premium shortfall on the term mortgages is likely to be offset by move-outs which exceed the predicted move-out rate once the monthly cash advances cease. Likewise, the excess premium on the tenure mortgages may be offset by move-outs at a lower rate than predicted.

The above comparison indicates that the principal limit method produces slightly higher monthly cash advances on term mortgages than does the basic model, and slightly lower cash advances on tenure mortgages. For the loan terms given in Tables 4 and 5, the differences in monthly advances are as follows:

	Prin Limit Method	Basic Model
10-Year Term	\$510	477
Tenure	357	372

The differences were considered for various ages, interest rates, and lengths of term. They were consistently found to be small. Furthermore, the likely effect on the move-out rates by type of payment pattern (i.e., more rapid move-outs upon expiration of term) made the differences seem negligible. When considered in conjunction with the advantages of simplicity, equity, and flexibility, the principal limit method was determined to be superior to the basic model for term and tenure mortgages.

III-F. Reserve for Administrative Costs and Risk

There is one final consideration to be discussed before moving on to the next section of this paper. This is an analysis of the premium reserve that will result when borrowers fail to utilize their maximum borrowing authority. The basic model can be used to analyze the botttom line present values of total expected premium and total expected losses for cash advance patterns that are below the maximums just as the maximums were analyzed in Tables 4 and 5.

The premium reserve can be generated in either of two ways. The first is when a borrower requests less than the maximum monthly cash advance that his or her principal limit will support. In such cases, the borrower in effect splits the principal limit into two accounts: one to use for the monthly cash advances, and one as a line-of-credit to hold the remaining principal limit. The second is when a borrower enters the HECM program with a property which is valued higher than the maximum FHA loan limit for the area. In the latter situation, the borrower's principal limit is calculated as if the property value were equal to the FHA maximum, ignoring the excess value. In the former case, there will be excess premium collected to the extent that the borrower fails to draw down the small line-of-credit account. In the latter, the excess property value will reduce future expected losses regardless of the extent of the borrower's utilization of principal limit.

Tables 6 and 7 illustrate the sensitivity of premiums and losses to changes in utilization of principal limit or excess property value. Essentially, the conclusion to be drawn from these tables is that the reserve premium collection is relatively large for relatively small amounts of underutilization or excess value. Furthermore, it will be reasonable to expect that an insurer's book of business will include a significant number of cases in which some underutilization or excess value exists.

The information presented in Tables 6 and 7 requires some elaboration. Similar to the presentation in Tables 4 and 5, it is based on the principal limit method for determining cash advances and on the basic model for computing expected premiums and losses of these cash advances. Both tables are explained in greater detail below.

Table 6 focuses on the first source of premium reserve collection: the level of utilization of principal limit by borrowers. The first column is a measure of borrower utilization. It assumes that cash advances will be taken as a monthly tenure payment. Thus the first column of the table lists utilization at 100, 95, and 90 percent of the maximum tenure payment determined by the principal limit method. The dollar amounts of the monthly tenure payments are listed in the rows labelled "payment". The rows marked "initial LOC" show the largest amount of initial principal limit that could be placed into a line-of-credit account while still supporting the indicated tenure payment. These calculations come directly from equations 10 through 12. Table 6 then shows for each utilization level the present value dollar amounts of expected MIP and expected losses, and the ratio of losses to MIP. The expected MIP and losses are computed from the basic model for the indicated tenure cash advance payments. It is assumed that the line-of-credit accounts are never used by the borrower. Note that the expected losses and premium for a 75 year old borrower at 100 percent utilization are the same as those given at the bottom of Table 5. The ratio of losses to premium for this borrower and utilization level is 90 percent. If, however, the same borrower utilizes only 95 percent of the maximum tenure payment, the loss to premium ratio drops to 79 percent. At 90 percent utilization, the ratio falls even further to 68 percent. Clearly, the reserve premium rises rapidly with small reductions in utilization.

Table 7 is similar in format to Table 6, except that it adds the additional effect of the initial property value exceeding the FHA Section 203(b) maximum loan limit by 10 percent. Note that the payment amounts, initial line-of-credit amounts, and the present value of expected MIP are all identical to those in Table 6. The reason is that the excess property value assumed in Table 7 does not affect the calculation of the borrower's principal limit, maximum tenure payment, or cash advance pattern. It only affects the calculation of expected losses. Note that the ratio of expected losses to expected MIP for a 75 year old borrower at 100 percent utilization falls from 90 percent in Table 6 to 73 percent in Table 7 due entirely to the 10 percent excess property value. Thus a small increase in excess property value also produces a significant amount of reserve premium.

What is the likelihood of the insurer's book including cases which involve underutilization or excess value? It is considerable. With regard

Table 6

PRESENT VALUES OF EXPECTED LOSSES/MIP COLLECTED
AT SELECTED UTILIZATION LEVELS

HOUSE VALUE EQUALS SECT. 203(B) LIMIT

Requested Tenure Payment--Percent of Max. Payment	Item	- - Age of Borrower - - 65	75	85
100 %	Payment	$218.13	356.61	607.08
	Initial LOC	$0	0	0
	Expected Loss	$3860	2880	1859
	Expected MIP	$3667	3201	2706
	Loss/MIP %	105%	90%	69%
95%	Payment	207.22	338.78	576.73
	Initial LOC	1226	1906	2770
	Expected Loss	3420	2486	1552
	Expected MIP	3599	3151	2675
	Loss/MIP %	95%	79%	58%
90%	Payment	196.32	320.95	546.37
	Initial LOC	2451	3811	5541
	Expected Loss	3005	2121	1277
	Expected MIP	3532	3100	2644
	Loss/MIP %	85%	68%	48%

Assumed Loan Terms:

House Value	=	$100,000
Sect. 203(b) Lim	=	100,000
Interest Rate	=	10%
Closing Costs	=	1,500
Servicing Fee	=	0
Initial Draw	=	0

Losses calculated assuming Line of Credit (LOC) is never used.

Table 7

PRESENT VALUE OF EXPECTED LOSSES/MIP COLLECTED
AT SELECTED UTILIZATION LEVELS

HOUSE VALUE 10 PERCENT OVER SECT. 203(B) LIMIT

Requested Tenure Payment--Percent of Max. Payment	Item	Age of Borrower 65	75	85
100 %	Payment	$218.13	356.61	607.08
	Initial LOC	$0	0	0
	Expected Loss	$3263	2333	1420
	Expected MIP	$3667	3201	2706
	Loss/MIP %	89%	73%	52%
95%	Payment	207.22	338.78	576.73
	Initial LOC	1226	1906	2770
	Expected Loss	2876	1999	1172
	Expected MIP	3599	3151	2675
	Loss/MIP %	80%	63%	44%
90%	Payment	196.32	320.95	546.37
	Initial LOC	2451	3811	5541
	Expected Loss	2514	1693	952
	Expected MIP	3532	3100	2644
	Loss/MIP %	71%	55%	36%

Assumed Loan Terms:

House Value	=	$110,000
Sect. 203(b) Lim	=	100,000
Interest Rate	=	10%
Closing Costs	=	1,500
Servicing Fee	=	0
Initial Draw	=	0

Losses calculated assuming Line of Credit (LOC) is never used.

to utilization, we note that there will be incentives for borrowers to set up lines-of-credit along with monthly cash advances rather than take the highest monthly cash advance to which they would be entitled. These incentives include equity preservation, the desire to maintain a line-of-credit for emergencies, and the treatment of reverse mortgage cash advances by certain entitlement programs for the elderly. The equity preservation argument is derived from the risk aversion that borrowers have with regard to outliving one's assets. The line-of-credit for emergencies is a particularly prudent option for borrowers with few other assets (HUD approved counsellors have been trained to recommend this to all borrowers). Finally, borrowers will need to manage their liquid assets so as to preserve their eligibility for certain entitlement programs (Medicaid and Supplemental Security Income, for example). Unless the cash advances are being spent in the month in which they are received, the borrower may run the risk of entitlement benefit reduction or even denial by taking cash advances that exceed his or her immediate need. Furthermore, negative arbitrage on reinvestment will usually make it economically infeasible for a borrower to draw cash advances that are not spent.

With regard to excess property value, we note that the FHA limits approximate 95 percent of median house price in a market area. In practice, the minimum of $67,500 and the maximum of $124,875 in high cost areas causes the standard to vary somewhat. Regardless of the exact percentile that the FHA limit represents in any particular market, properties within the FHA limit are generally the lower end of the local market. As a result there are many potential borrowers whose homes will be above the limit. Until such time as there is a competitive market for privately insured or uninsured reverse mortgages to serve these borrowers, the HECM program may be attractive to them, the FHA limit notwithstanding. Therefore, it is resonable to expect a significant book to be written on properties with excess value. When combined with the underutilization effects described above, there should be sufficient reserve premium to pay for administrative expenses and maintain a risk contingency capital reserve.

IV. Actuarial Assumptions and Parameter Sensitivity

IV-A. Introduction

The actuarial assumptions in the HECM payments model will be separated into two groups for discussion. The first are the assumptions regarding the survival probabilities of borrowers. These are the $S_{i,j}$ from equation (3). The second involve the remaining model parameters which must be estimated by the insurer. The latter consist of the annual average appreciation rate (μ), the standard deviation of annual apreciation rates about the average (σ), the move-out rate (m), and the discount rate (i).

Subsection IV-B deals with the issues involving the survival probabilities of HECM borrowers. Subsection IV-C deals with the estimation of the other model parameters.

IV-B. Survival Probabilities

The issues involved in the treatment of borrower mortality include (1) the effect of possible adverse selection on the choice of mortality table, (2) the trends in future mortality rates due to health care improvements, and (3) the issue of whether to adopt joint mortality tables for co-borrowers. The sensitivity of the model to changes in the underlying mortality assumption is dealt with in IV-C in which changes to the move-out rate are analyzed.

1. Adverse Selection

The issue of adverse selection in reverse mortgage programs is a difficult one to address. Data from private reverse mortgage programs with regard to the mortality rates of borrowers is proprietary information and not generally available. Data from private as well as public sponsored programs (State agencies or non-profit groups), where available, are not very useful in assessing the possible adverse selection in the HECM program. The reasons are differences in program design[27], rapid prepayments in these programs which limits data on mortality[28], and lack

[27] None of the public sponsored programs offer a tenure payment option. There is little incentive for borrowers who expect to live a long time to adversely select a program which offers only fixed term mortgages of 10 to 12 years maximum, because any borrower who expects to live longer than the maximum fixed term runs a great risk of outliving his or her assets by entering such a program. Such borrowers may explore other options.

[28] A Minnesota lender offered an 8 to 12 year fixed term reverse mortgage. Anecdotal evidence obtained from the lender indicates that rapid prepayment has resulted in an average mortgage term of 3 years. Many borrowers were in poor health and sought reverse mortgages as temporary financial relief. Others used the reverse mortgages to finance housing searches in and subsequent moves to warmer climates. Very little mortality experience was obtained. Another program, offered by the San Francisco Developmant Authority, was designed as fixed term with 10 year maximum.

of sufficient numbers of borrowers to construct program-specific mortality tables.

Absent data from other reverse mortgage programs to address the adverse selection question, we looked at how the problem is treated in a complementary product, specifically, continuing care retirement communities (CCRC's). According to Winklevoss and Powell (1984), CCRC's are organizations established to provide housing and services, including health care, to people of retirement age. The communities are often campus-like settings that offer both independent living arrangements and congregate living arrangements for those needing a greater level of health care services. CCRC's offer the residents the guarantee of shelter and various health care services for life. Like the HECM program, the minimum age requirement for entering a CCRC is 62. However, the fee structure includes a large one-time entry fee and an additional monthly fee. Winklevoss and Powell found that fee structures varied considerably by quality and level of housing and services provided, and that the average at the time of their study was $34,689 up-front and $562 monthly for one person, and $38,682 plus $815 monthly for a couple. Clearly, the typical CCRC program will be most attractive to persons who expect to live a long time, and who are seeking a great deal of protection from the possibility of outliving their assets.

How then do managers of CCRC's price their product when the extent of adverse selection is unknown? Each such community is likely to attract residents with unique characteristic profiles depending on how the overall package of housing and services provided by the community compares to alternatives available in the market. Ideally, if the CCRC were large enough, the management would construct a mortality table based upon the age-specific mortality rates observed at the facility over time. Few projects have enough data to accomplish this. An alternative is often used and it is called the "standardized mortality ratio" approach. In this technique, the management selects a "base" mortality table from which to construct a "decrement" table for the community. The decrement table represents the annual expected number of deaths in the community based upon the population age and sex distribution and the mortality rates in the base table. Then, as a sufficient time period elapses (say five years), actual decrements are compared to the expected decrements in the table, and decisions can be made as to the need to adjust the actuarial assumptions due to adverse selection.

For CCRC's using the standardized mortality ratio method, the base table recommended by Winklevoss and Powell is an insurance company annuitant mortality table. Rates in such a table are appropriate because both purchasers of private annuities and continuing care retirement contracts are willing to make substantial up-front financial commitments to protect themselves from outliving their assets. Neither would be likely to

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Similar to the Minnesota experience, there were many prepayments in the first two years. No mortality data is available for those who did not prepay.

purchase these contracts if they did not believe they were in good health and likely to live longer than the average life expectancy for their attained age.

The lack of program experience from which to determine borrower mortality rates and to assess the adverse selection question in the HECM program will require the use of an approach similar to the standardized mortality ratio technique described above. The use of an annuitant mortality table as the base, however, seemed inappropriate for several reasons. The first is that the up-front financial commitment to enter the HECM program is much smaller than that of a comparable annuity or CCRC contract[29]. The second is that the program has been designed to allow borrowers who are in poor health to receive the full amount of their principal limit in the early years of the loan when needed, rather than require them to enter a tenure plan or term plan which defers payments into the future. Thus, an elderly homeowner in poor health may access the relatively large sums of cash necessary to supplement Medicare/Medicaid coverage for major medical treatment or for in-home nursing care through the HECM program. Of course, the HECM program may also attract those borrowers who believe themselves to be in above average health too, particularly because of the borrower's option to receive cash advances as a tenure payment. Low entrance costs plus the flexibility to receive cash in the present as well as deferred into the future are likely to make the HECM program attractive to both kinds of borrowers: those in good health and those in poor health. If program selection follows this logic, then the mortality rates averaged for all borrowers could approximate those of the general population, which also includes elderly persons in good health and those in poor health[30]. For these reasons, the general population

[29] While up-front loan costs will vary by state and lender in the HECM prorgam, we expect that they will average about 3 to 5 percent of the maximum claim amount. This amounts to $3,000 to $5,000 on a $100,000 home. The estimate includes the up-front MIP charge, the lender's origination fee, appraisal fee, title insurance, and other miscellaneous loan costs.

[30] If we define elderly persons in poor health as those whose age-specific mortality rates are double those of the general population, and elderly persons in good health as those whose age-specific mortality rates are given by an insurance annuity table (source: Society of Actuaries, 1983), then the blended mortality of a population whose members were initially chosen in equal numbers from both groups gives a life expectancy that is comparable to that of the general population. For example, the following are the life expectancies of a 65 year old female:

Population	Life Expectancy (Years)	
A. General Population (100% 1979-81 U.S. mortality)	Average Median	18.4 18.8
B. "Poor Health" Group (200% 1979-81 U.S. mortality)	Average Median	13.3 13.3

mortality table has been chosen as the base table for HECM. After the passage of some time, mortality experience from the program will become available. Adverse selection will then be analyzed by comparing expected decrements derived from the base table to actual decrements observed.

2. Mortality Trends

Improvements in health care, lifestyle changes, and environmental changes are factors which can affect the life expectancies of the general population. We are particularly interested in the life expectancy trends of current elderly cohorts in the general population, from which HECM borrowers will come. Furthermore, we are interested in knowing whether the life expectancy trends of HECM borrowers will follow the same trends as the general population, or whether the self selection process of the program will systematically bias the life expectancies of borrowers.

The most important factor influencing future life expectancies of the elderly are medical advances such as the development and application of new diagnostic, surgical, and life-sustaining techniques. Such improvements are likely to reduce future age-specific mortality rates in the general population; however, the increased expense of health care may result in less benefit to likely HECM borrowers who are probably the least able to afford these services.

Lifestyle changes that may affect the elderly are the future use of tobacco and drugs (prescription drugs and alcohol, in particular), dietary improvements, and intangible factors which determine the quality of life and how the elderly perceive their value to society. Participation in the HECM program may actually improve the quality of life for many borrowers, thereby increasing their life expectancy merely through participation[31]. Thus, the mortality trends of the population served by a particular program may differ from those of the general population.

Environmental factors affecting the elderly are air and water quality, as well as the cumulative effects of long-term exposure to harmful substances, which may have gone unrecognized in the past. It is possible that environmental factors will partially offset gains in life expectancy among the elderly attributable to health care and lifestyle improvements.

C.	"Good Health" Group (100% 1983 annuitant mortality)	Average Median	21.8 22.7
D.	Blended (B + C) (Equal weight to both groups)	Average Median	17.5 17.6

[31] Profound improvements in life expectancy are expected at continuing care retirement centers, which include health care as part of the product.

We can determine no reason why HECM borrowers should be affected differently than the general population in this area.

The Social Security Administration has made projections of life expectancy and mortality rates for many years into the future as a part of its actuarial analysis of the Social Security Trust Fund. Tables 8 and 9 illustrate the projected trends in the general population. Table 8 presents three projection scenarios of the life expectancy of a 65 year old (male and female tables) as it would be calculated in each calendar year shown. Note that in 1980 a 65 year old male had a life expectancy of 14.0 years, while a 65 year old female had a life expectancy of 18.4 years. These estimates are consistent with the 1979-81 U.S. Decennial Life Tables. By 1990, however, the agency projects that these life expectancies will have increased to between 15.0 and 15.2 years for a 65 year old male, and 18.8 to 19.1 years for a 65 year old female. By 2000, the life expectancies are projected to be flat under alternative 1, and to increase even more under alternatives 2 and 3. Table 9 presents mortality trends in a different manner than Table 8. The differences are that Table 9 applies only to females, it lists central death rates rather than life expectancies, and the projections follow a cohort of elderly females into the future rather than showing the projections for a single attained age.

The implications for the HECM program are as follows. It is reasonably clear that life expectancies in the general population will rise over time. What is not clear is the extent to which HECM borrowers will reflect the general population shift. The likely appeal of the program to those in poor health as well as others may result in less improvement in life expectancy than the general population. Furthermore, the extent to which HECM will appeal to very low income borrowers may mean that borrowers receive less health care on average than others. If so, they may not benefit from the new health care procedures as much as the general population. On the other hand, the extent to which HECM cash advances are used to pay for additional health care and overall lifestyle improvements may offset the low income effect. In conclusion, we note that, like the issue of adverse selection, mortality trending of HECM borrowers is a phenomenon that will have to be observed. As such, the the base mortality table of the program was not adjusted for trending.

3. Survivorship for Co-borrowers

Given two individuals, such as a husband and wife, two sisters, or two brothers (all are possible co-borrower situations in HECM), many questions may be asked regarding their joint survivorship probabilities. This paper addresses some of these questions. First, what is the probability that at least one co-borrower will survive for a given number of years into the future? How does this compare to the probability that the younger of the two will survive for the same number of years? The joint survivorship is likely to be higher than that of the individual, but by how much? Second, what is the average number of years that we would expect at least one of the two to survive? How does this average compare to the average life expectancy of the younger borrower? The average for

Table 8

TRENDS IN LIFE EXPECTANCY[1] AT AGE 65 THROUGH THE YEAR 2000
BASED ON OBSERVED AND PROJECTED AGE-SPECIFIC MORTALITY RATES
(SOURCE: SOCIAL SECURITY ADMINISTRATION)

Calendar Year	--Observed---		---Projected[2]---					
			Alternative 1		Alternative 2		Alternative 3	
	Male	Female	Male	Female	Male	Female	Male	Female
1980	14.0	18.4[3]	–	–	–	–	–	–
1982	14.5	18.8	–	–	–	–	–	–
1984	14.4	18.7	–	–	–	–	–	–
1986	14.5	18.7	–	–	–	–	–	–
1988	14.9	18.8	–	–	–	–	–	–
1990	–	–	15.0	18.8	15.1	19.0	15.2	19.1
1992	–	–	15.0	18.8	15.2	19.1	15.4	19.4
1994	–	–	15.0	18.9	15.3	19.3	15.7	19.7
1996	–	–	15.0	18.9	15.4	19.4	15.9	19.9
1998	–	–	15.0	18.9	15.6	19.5	16.1	20.2
2000	–	–	15.0	18.9	15.6	19.6	16.2	20.4

[1] Life expectancy is defined as the average remaining years of life to a 65 year old person if he or she were to experience the age-specific mortality rates for the tabulated year throughout the remainder of his or her life.

[2] The Social Security Administration makes three alternative projections about changes in overall age-specific death rates by analyzing trends in ten specific causes of death. These causes are: heart disease, cancer, vascular disease, violence, respiratory disease, infancy, digestive disease, diabetes mellitus, cirrhosis (liver), and other.

[3] This is the life expectancy, in years, for all 65 year old borrowers in the HECM model. It is based on the observed mortality rates in the 1979-1980 U.S. Decennial Tables.

Table 9

Central Death Rates Per 100,000 Females Projected by Year and Attained Age

Source: Social Security Administration

Year/Attained Age	W/O Projection	- Alternative Projections - Optmis.	Likely	Pessim.
1985 / 70-75	2609	2609	2609	2609
1990 / 75-80	4108	3898	3823	3749
1995 / 80-85	6717	6579	6193	5857
2000 / 85-90	11264	10670	9637	8751
2005 / 90-95	18116	17549	15732	14161

Note: Central death rate is the ratio of the number of deaths during the year to persons at the tabulated age to the midyear population at that age. These rates are then multiplied by 100,000 to give the table entries.

co-borrowers is likely to be greater than that of the individual, but by how much? Finally, what is the likely change in the move-out rate? Is the move-out rate higher for an individual borrower because co-borrowers are less likely to seek nursing home care while one remains well enough to care for the other? Or, is the move out rate higher for a couple because the survivor is more likely to move (for physical reasons such as difficulty in maintaining the house alone or for emotional reasons) soon after a spouse dies or moves to a nursing home ?

The answer to the first question depends upon the age difference of the co-borrowers. The probability of at least one borrower surviving for a period of time (say 10 years) is greatest when the older borrower's age is equal or nearly equal to that of the younger. Table 10 illustrates these probabilities for various combinations of borrower age and gender, with the younger borrower assumed to be 75 years old. Note that as the older borrower's age increases, the joint survival probability approaches that of the younger borrower, taken individually. For example, if a husband and wife were both age 75, then there would be a 73.2 percent probability that at least one of them would be alive ten years from now, based on the 1979-81 U.S. Decennial Life Tables[32]. However, if the husband's age were 85 instead of 75, the probability drops to 61.7 percent. By herself, the 75 year old woman has a 56.2 percent chance of surviving 10 years.

The answer to the second question, as the first, also depends on the age difference of the co-borrowers. The average number of years that at least one borrower will survive is greatest when the older borrower's age is equal or nearly equal to that of the younger. This average for 75-year old co-borrowers is about three to four years longer than the average life expectancy of an individual 75-year old female[33]. The difference decreases as the age difference between co-borrowers increases.

Unfortunately, there is no good answer to the third question dealing with the change, if any, in the move-out rates when co-borrowers are

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[32] To derive this number, let ${}_{10}P_{F75}$ be the 10 year survival probability of a 75 year old female, and ${}_{10}P_{M75}$ be the 10 year survival probability for a 75 year old male. Then the 10 year joint survival probability is given by:

$${}_{10}P_{F75,M75} \quad = \quad {}_{10}P_{F75} + (1 - {}_{10}P_{F75}) \text{ X } {}_{10}P_{M75}.$$

In the example, ${}_{10}P_{F75} = .562$, and ${}_{10}P_{M75} = .389$ (source: 1979-81 U.S. Decennial Life Tables). Hence the joint probability is:

$${}_{10}P_{F75,M75} \quad = \quad .562 + (1 - .562) \text{ X } .389 \quad = \quad .732.$$

[33] The life expectancy of a 75 year old female is 11.4 years. For two female co-borrowers, both age 75, the average number of years that at least one will survive is 15.1. If one of the co-borrowers is male, the average is reduced by about one year.

involved. Such changes will be observed from program experience as part of the demonstration.

The HECM payments model uses the female general population mortality table for all borrowers, including co-borrowers. The reasons are as follows. The majority of borrowers is expected to consist of single females[34]. Couples should be the next largest group, followed by single males. The potential underestimation of life expectancy for couples will be offset at least partially in two ways. The first is that single male borrowers have life expectancies three or four years less than that of females of the same age (see Table 8), which is approximately equal to the underestimation for a couple. The second is that some couples are likely to move out at an accelerated rate after the death of one spouse, resulting in reductions in loan survival probabilities. The combination of the two effects, along with the expected dominance of single females as borrowers justifies the use of the single mortality table for all borrowers. The benefit of simplicity in not having to develop separate principal limit factors for co-borrowers of various age combinations was also considered in making this decision.

IV-C. Parameter Estimation

The main issues remaining with regard to the model's actuarial assumptions pertain to the estimation of the mean and variance of the stochastic process used to simulate future property values. The choice of the other two parameters, namely the move-out rate and the discount rate, has been discussed previously in the text, and those discussions will not be repeated here. However, the model's sensitivity to changes in all four of these parameters will be addressed below in order to assess the impact of any errors in parameter estimation.

1. Mean Appreciation

Figure 3 shows that there has been an historically close relationship between the average annual house price appreciation as determined by a constant quality housing price index[35] and the overall inflation rate as determined by the consumer price index. Both housing appreciation and overall inflation averaged slightly over 6 percent per year between 1977 and 1988. For this period, regression analysis confirms that there was an

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[34] Weinrobe (1988) in his study of existing reverse mortgage programs found the predominance of single females to be consistent in all the programs for which data were available. He reports the average distribution to be: 63 percent single females, 12 percent single males, and 25 percent married couples.

[35] This index is based upon average sales prices of the kinds of houses sold in 1982. These are estimated using hedonic regression analysis of new construction house price data gathered by the Census Bureau. For an explanation of the methodology used, see Current Construction Reports, Price Index of New One-Family Houses Sold, July 1989, Bureau of the Census.

Table 10

Illustration of Joint Mortality

Probability that at least one individual will survive for 10 years.

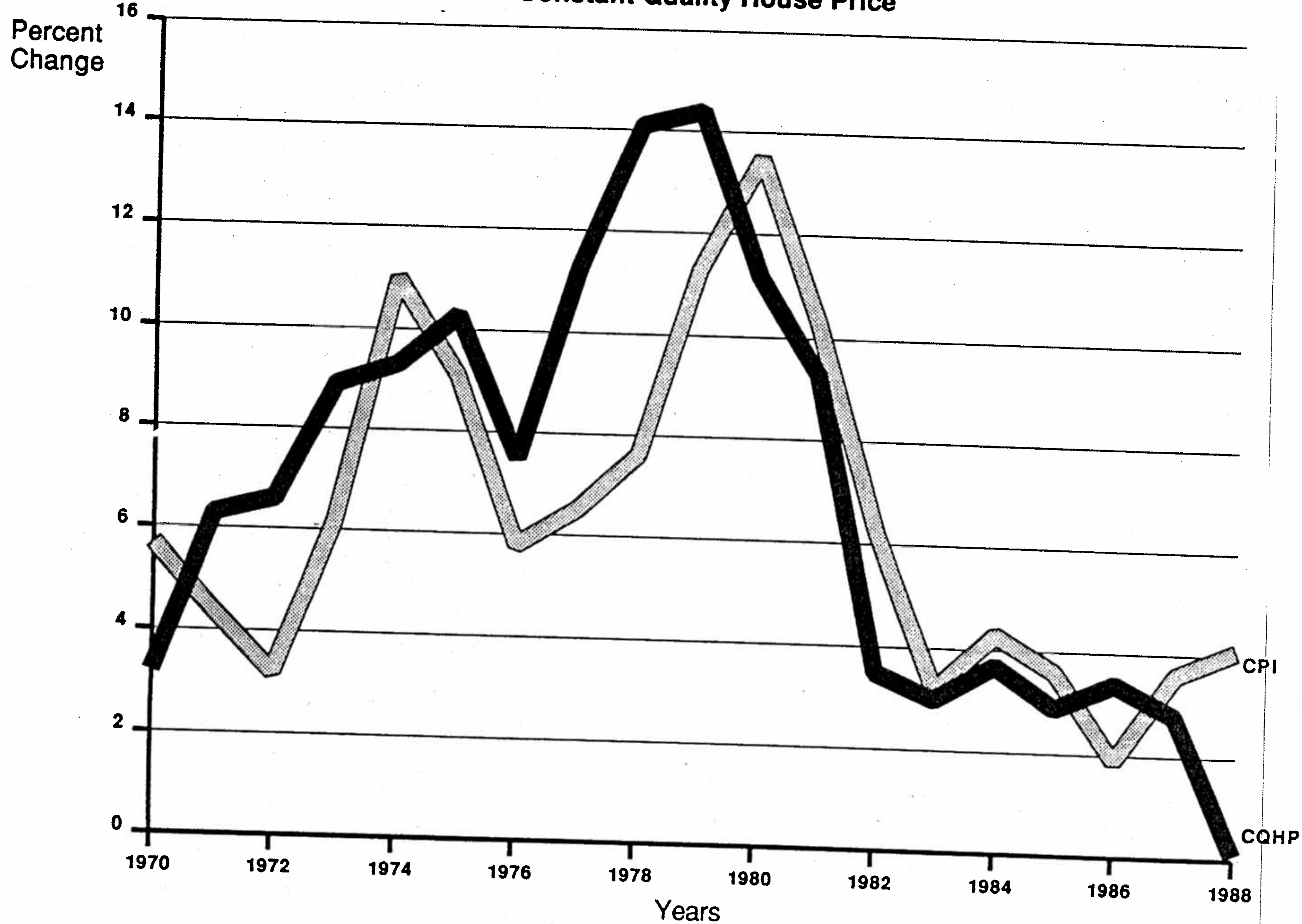
75 Year Old Female with:	10 Year Survival Probability [a]
Male age 75	.732
Female age 75	.808
Male age 80	.668
Female age 80	.728
Male age 85	.617
Female age 85	.653
Male age 90	.583 [b]
Female age 90	.602 [b]
No co-borrower	.562
75 Year Old Male with:	
Male age 75	.627
Female age 75	.732
Male age 80	.537
Female age 80	.619
Male age 85	.466
Female age 85	.516
Male age 90	.418 [c]
Female age 90	.446 [c]
No co-borrower	.389

[a] Survival probabilities have not been adjusted for move-outs.

[b] If terminal age of 100 is assumed, then the 10 year joint survival probability becomes .562, same as if there were no co-borrower.

[c] If terminal age of 100 is assumed, then the 10 year joint survival probability becomes .389, same as if there were no co-borrower.

Figure 3
Changes in Consumer Price Index and Constant Quality House Price



approximately one-to-one relationship between average annual changes in constant quality house prices, and overall prices. The mean appreciation rate parameter in the model (μ) is a nominal appreciation rate, and not a real rate. Thus, a continuation of the observed one-to-one relationship with the CPI along with the model's assumption of a 4 percent mean housing appreciation rate implies an economic environment in which the CPI is expected to increase by 4 percent annually.[36] This is not an unreasonable economic environment for the 1990's. Even if housing inflation lags the CPI for a few years (as has been the case the last 3 years), the actuarial soundness of the program is not jeopardized. HECM insurance claims are expected to be "back loaded". This means that nominal housing inflation below 4 percent in the early years can be offset by nominal inflation above 4 percent in the later years without substantial losses being incurred.

Other researchers have compared constant quality housing price indices with the CPI for individual metropolitan area markets. Case and Schiller (1987) constructed their own housing index for four metropolitan areas using a technique called the weighted repeat sales method[37]. The areas were Atlanta, Chicago, Dallas, and San Francisco, and the time period covered by their study was 1970 through 1986. When they compared the results to the CPI, they found that constant quality house appreciation in all four cities was greater than or equal to the CPI change during the same time period[38].

One final question arises with regard to the choice of a mean appreciation parameter. It is whether houses occupied by elderly homeowners will appreciate as much as those occupied by homeowners of all ages? Some have argued that the elderly live in older homes located in older neighborhoods, and therefore, house appreciation will lag behind that of the general population. We have not found any study that would confirm this. In fact, we have produced some evidence that would indicate that the difference, if it exists at all, may not be very large. Specifically, we examined the national longitudinal sample of the Annual Housing Surveys from 1974 and 1983. Using the technique described in the next subsection of this paper, we estimated the average annual mean and variance of appreciation based on respondant's self reported value estimates of the

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[36] If one accounts for depreciation of existing homes at a rate of between ½ and 1 percent per year, then average overall inflation of 4½ to 5 percent per year is necessary to support the model's 4 percent expected housing appreciation estimate.

[37] This method utilizes large data bases on real estate transactions in a metropolitan area to identify single family homes which sold more than once during the period covered by the data. Then, using only the repeat sales to construct the price index, the authors avoid the quality biases inherent in other housing price indices (such as median sales prices reported by the National Association of Realtors).

[38] Expressed as average annual percentage changes in the indices between 1970 and 1986, Case and Schiller report the following nominal housing appreciation rates: Atlanta 6.9%, Chicago 7.0%, Dallas 9.1%, and San Francisco 11.3%. During the same period the CPI averaged 6.7% per year.

same houses in both surveys. In the first case, houses were selected if they had a mortgage and an initial value greater than $20,000 in 1974[39] without regard to the age of the borrowers. The annualized mean and variance of the appreciation rates of these houses was computed to be .083 (8.3 percent) and .017, respectively. In the second case, houses were selected as before, but with the additional requirement that the homeowner in 1974 be in the 55 or older age category[40]. The second sample had annualized mean and variance of appreciation of .076 and .016. This finding, while not a full study of the question, did reassure us that houses occupied by the elderly do not have appreciation characteristics that are vastly different from those of all houses.

2. Variance of Appreciation

Most readers know what is a reasonable estimate for a mean appreciation rate based upon their general knowledge of the housing market. The assumed value of 4 percent annual appreciation in the model can be compared to the 1970's and 1980's when the rate averaged around 7 or 8 percent. The 1990's are likely to produce lower appreciation; hence, the 4 percent estimate seems reasonable. However, few readers are likely to know what is a reasonable estimate for the variance or standard deviation of appreciation based upon a general knowledge of the market. Even experts in the housing field have not determined how to measure this parameter[41].

The model uses an assumed variance (σ^2) of .01, which is equivalent to a standard deviation (σ) of .10 (10 percent). We have not determined a method of measuring this parameter precisely. Instead, we have used a technique which makes use of the national longitudinal sample of the Annual Housing Surveys between 1974 and 1983 to estimate the order of magnitude of the variance. This technique will be described below, and the results presented in Table 11.

We call the technique the "mid-point" approach. It provides estimates of both the mean (μ) and variance (σ^2) of appreciation averaged over a multi-year period. The problem with using AHS data to estimate μ and σ^2 for short periods of time--i.e., one or two years--is that the survey respondants' estimates of house values are reported by class interval rather than point estimates. If one allocates the house value to be the

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[39] Choice of houses with a mortgage is a proxy for standard quality, while the $20,000 minimum value is a way of eliminating "handyman specials" which may have undergone substantial upgrading between the survey dates.
[40] One could argue that many elderly homeowners have paid off their mortgages; hence, they would not be included in the second sample. In response to this, we note that HECM borrowers would all occupy homes which meet the quality standards to obtain a mortgage or else they would be required to bring the property up to standard as a condition of the HECM loan agreement.
[41] Previously cited studies which model housing assets using a similar geometric Brownian motion process do not indicate how the parameter (σ) is to be measured.

mid-point of the reported class interval, then substantial errors could be introduced, particularly when estimating the volatility parameter. To minimize errors, we extend the time period. Two multiple year time periods were examined: 1974 to 1983 (9 years) and 1978 to 1983 (5 years). The nine year period is the longest period for which the longitudinal sample could be followed, while the shorter period was chosen to eliminate some of the high housing inflation years of the mid-70's.

The mid-point technique begins with the assumption that house prices follow a geometric Brownian motion process as defined previously. This means that over a multi-year period of t years the distribution of the cumulative appreciation rates should have a mean of μt, a variance of $\sigma^2 t$, and standard deviation of $\sigma\sqrt{t}$. The sample of houses to be selected included only those with mortgages and initial values greater than $20,000 in the base year. Let

H_{78} = mid-point value estimate of a selected house in the 1978 survey, and

H_{83} = mid-point value estimate of the same house in the 1983 survey.

Then $\ln(H_{83}/H_{78})$ is one observation of the random variable representing the 5 year cumulative appreciation rate. We compute the mean, variance, and standard deviation of all such observations in the sample and divide by t or the square root of t, as appropriate, to obtain annualized parameter estimates. The results of our calculations are presented in Table 11. Based on these somewhat imprecise measurements, we concluded that the order of magnitude of the variance parameter is about .01. This is the value that we assumed in the HECM model.

3. Sensitivity to Parameter Changes

The final consideration in this section is an analysis of the sensitivity of the model to changes in the parameters. As mentioned previously, the sensitivity to changes in the underlying mortality rates are not explicitly shown here. Instead, the sensitivity to changes in the move-out rate parameter are shown. Together the mortality and move-out rates define the rate of mortgage prepayment, which is the important phenomenon that these parameters are attempting to quantify. Lower mortality can be offset by higher move-outs and vice versa. Hence, the sensitivity to prepayment speed is shown by variations in the assumed move-out rate parameter.

Table 12 is similar in format to Tables 6 and 7. The loan terms are calculated using the principal limit method of the current model. The expected losses and MIP collection are then analyzed by using the assumed parameter values in the basic model.

From the table, it is clear that the HECM model is very sensitive to small changes in the mean appreciation rate. An average appreciation rate

Table 11

Appreciation Characteristics by Type of Mortgage[1]

	All Houses in Sample		Houses Sold During Base Year[2]	
	FHA	Non-FHA	FHA	Non-FHA
1978-83				
Mean Appreciation	.071[3]	.065	.064	.060
Variance	.013	.015	.009	.011
Standard Deviation	.113	.123	.093	.103
1974-83				
Mean Appreciation	.083	.084	NA	NA
Variance	.017	.017	NA	NA
Standard Deviation	.128	.131	NA	NA

[1] Source: National Longitudinal Sample of the Annual Housing Surveys for 1974, 1978, and 1983.

[2] Restricting the sample to houses sold during the base year may improve the accuracy of the reported values, which are estimates solicited from the survey respondants. The reason is that respondants are likely to have more accurate knowledge of the value of their house in the year of sale. Insufficient cases exist in the sample to restrict it to houses sold in both years.

[3] 7.1 percent continuously compounded annual rate.

Table 12

SENSITIVITY OF PRESENT VALUES OF EXPECTED LOSSES/MIP COLLECTED TO CHANGES IN THE MODEL PARAMETERS

Assumed Loan Terms:

Borrower Age	=	75	Closing Costs	=	$1,500
Max Claim Amt	=	$100,000	Initial Draw	=	0
Interest Rate	=	10%	Tenure Payment	=	$357
Servicing Fee	=	0	Line of Credit	=	0

Parameter	Lower	Model Assumption	Higher
Mean Appreciation (μ)	3%	4%	5%
Expected Loss	$4030	2880	1904
Expected MIP	3201	3201	3201
Loss/MIP %	126%	90%	59%
Apprec. Variance[1] (σ^2)	.005	.010	.015
Expected Loss	2545	2880	3168
Expected MIP	3201	3201	3201
Loss/MIP %	80%	90%	99%
Move-Out Rate (m)	0.0	0.3	0.6
Expected Loss	4424	2880	1938
Expected MIP	3481	3201	3005
Loss/MIP %	127%	90%	64%
Discount Rate (i)	8.5%	9.5%	10.5%
Expected Loss	3486	2880	2384
Expected MIP	3319	3201	3098
Loss/MIP %	105%	90%	77%

[1] Corresponding standard deviations (σ) are 7 percent, 10 percent, and 12¼ percent, respectively.

that varies by one percent in either direction from the assumed 4 percent causes the ratio of loss to MIP to range from 126 percent down to 59 percent. The model appears to be somewhat less sensitive to changes in the variance, although it is difficult to determine whether the range of values shown for the variance represent a small or a large deviation from the assumed value. The model is quite sensitive to the changes in the move-out rate also. As with the variance, it is difficult to determine whether the range shown represents a small or large deviation for this parameter. For both the variance and the move-out rate, the data gathered from the demonstration should provide some indication as to the likely magnitude of fluctuations. Finally, the sensitivity to the discount rate is not great. Large deviations in the discount rate from the assumed value are unlikely because the rate assumed will necessarily be close to the value of the ten year Treasury rate.

Appendix 1

Derivation of Expected Value and Conditional Expected Value for Log-Normally Distributed House Prices

Let

$H(t)$ = house price at time t,
H_o = initial house price, $t = 0$,
$X(t)$ = $H(t)/H_o$,
$Y(t)$ = $\ln(H(t)/H_o)$,
$B(t)$ = outstanding balance on mortgage at time t, and
$b(t)$ = $B(t)/H_o$.

Then

$$X(t) = e^{Y(t)},$$

where we assume that Y(t) is a Brownian motion process with drift. Note that this assumption implies that for any given time $t = t^*$, $Y(t^*)$ is a normally distributed random variable with mean μt^* and variance $\sigma^2 t^*$, where μ and σ are constants. Furthermore, X(t) is a geometric Brownian motion and is a lognormally distributed random variable.

Assume:

time is fixed at $t = 1$, making the mean and variance of Y(t) equal μ and σ^2,
x is an observed value of the random variable X(t), and
y is an observed value of the random variable Y(t).

For simplicity of notation, we shall use the variables X, Y, and b in place of X(t), Y(t), and b(t), keeping in mind that the variables are actually time dependent. The assumption that $t = 1$ will be relaxed when appropriate. The problem is to find $E[X]$ and $E[X \mid x < b]$.

A. <u>Derive E[X]:</u>

Let

$$g(Y) = e^Y.$$

Then $E[X] = E[e^Y] = E[g(Y)]$. That is, finding the expected value of X is equivalent to finding the expected value of the function $g(Y) = e^Y$. According to Ross (1983) (chapt. 1.3), the expected value of the function of a random variable is given by:

$$E[g(Y)] = \int_{-\infty}^{+\infty} g(y)dF(y) = \int_{-\infty}^{+\infty} g(y)f(y)dy \; , \qquad \text{(A-1)}$$

where

F(y) is the probability distribution function of Y, and
f(y) is the probability density function of Y.

(Note that the integrals in (A-1) are non-stochastic because they are expressed in terms of the real-valued variable, y, and not the random variable, Y).

Since Y is assumed to have the normal distribution, we have:

$$f(y) = [1/\sigma\surd(2\pi)]\, e^{-\frac{1}{2}[(y-\mu)/\sigma]^2} \; , \qquad \text{(A-2)}$$

where equation (A-2) is the density function of a normally distributed random variable with mean μ, and variance σ^2. Substituting (A-2) into equation (A-1), we get:

$$E[g(Y)] = E[e^Y] = [1/\sigma\surd(2\pi)] \int_{-\infty}^{+\infty} e^y\, e^{-\frac{1}{2}[(y-\mu)/\sigma]^2}\, dy \; . \qquad \text{(A-3)}$$

Now let

$$y' = (y-\mu)/\sigma \; ,$$

which transforms the y into observed values of a standardized random variable. (Note that $dy' = dy/\sigma$). Rewrite equation (A-3) and simplify:

$$E[e^Y] = e^\mu\, [1/\surd(2\pi)] \int_{-\infty}^{+\infty} e^{\sigma y'} \cdot e^{-\frac{1}{2}y'^2}\, dy' \; ,$$

$$= e^{\mu+\frac{1}{2}\sigma^2}\, [1/\surd(2\pi)] \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(y'-\sigma)^2}\, dy' \; ,$$

$$= e^{\mu+\frac{1}{2}\sigma^2}\, \beta \; . \qquad \text{(A-4)}$$

As proved by Parzen (1960) (chapt. 2.24), β is an identity which equals 1. That is,

$$\beta = [1/\surd(2\pi)] \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(y'-\sigma)^2}\, dy' \equiv 1. \qquad \text{(A-5)}$$

Combine equations (A-4) and (A-5) to get:

$$E[X] = E[e^Y] = e^{\mu+\frac{1}{2}\sigma^2} \; . \qquad \text{(A-6)}$$

Since the variable X, and the constants μ and σ are actually time dependent, we rewrite (A-6):

$$E[X(t)] = e^{\mu t + \frac{1}{2}\sigma^2 t} \text{ , and}$$

$$E[H(t)] = H_0 e^{\mu t + \frac{1}{2}\sigma^2 t} \text{ .} \qquad \text{(A-7)}$$

B. Derive $E[X|x < b]$:

Again let

$$g(Y) = e^Y.$$

Then $E[X|x < b] = E[g(Y)|y < \ln(b)]$. That is, the conditional expected value of X given that the observed value, x, is less than b is equivalent to the conditional expected value of the function g(Y) given that y is less than the natural log of b. Rewrite equation (A-1):

$$E[g(Y)|y < \ln(b)] = \int_{-\infty}^{+\infty} g(y)d\Phi(y)$$

$$= \int_{-\infty}^{+\infty} g(y)\phi(y)dy \text{ ,} \qquad \text{(A-8)}$$

where $\Phi(y)$ and $\phi(y)$ are revised probability distribution and density functions. Specifically, the revised probability density function is given by:

$$\phi(y) = [1/A]\ [1/\sigma\sqrt{(2\pi)}]\ e^{-\frac{1}{2}[(y-\mu)/\sigma]^2}, \qquad \text{(A-9)}$$

if $-\infty < y \leq \ln(b)$, and

$$\phi(y) = 0 \text{ ,}$$

otherwise. The factor [1/A] is required so that the total area under the probability density function equals 1, as is required of all probability density functions. (Figures A-1 and A-2 illustrate the conditional and unconditional probability densities for the normal and log-normal distributions.) That is, in order for

$$\int_{-\infty}^{+\infty} \phi(y)\ dy = 1 \text{ ,}$$

then the constant A must be given by:

$$A = [1/\sigma\sqrt{(2\pi)}] \int_{-\infty}^{\ln(b)} e^{-\frac{1}{2}[(y-\mu)/\sigma]^2}\ dy. \qquad \text{(A-10)}$$

Therefore, from equations (A-8), (A-9), and (A-10) we have:

$$E[X|x < b] = E[e^Y|y < \ln(b)] =$$

$$[1/A]\ [1/\sigma\sqrt{(2\pi)}] \int_{-\infty}^{\ln(b)} e^y\ e^{-\frac{1}{2}[(y-\mu)/\sigma]^2}\ dy. \qquad \text{(A-11)}$$

Let

$$y' = (y-\mu)/\sigma \text{ , and}$$
$$U = [\ln(b)-\mu]/\sigma \text{ ,}$$

which transforms both y and ln(b) into standardized form. Rewrite equation (A-11) and simplify:

$$E[X|x < b] = e^{\mu} [1/A] \quad [1/\sqrt{(2\pi)}] \int_{-\infty}^{U} e^{\sigma y'} e^{-\frac{1}{2}y'^2} dy'$$

$$= e^{\mu+\frac{1}{2}\sigma^2} [1/A] [1/\sqrt{(2\pi)}] \int_{-\infty}^{U} e^{-\frac{1}{2}(y'-\sigma)^2} dy'$$

$$= e^{\mu+\frac{1}{2}\sigma^2} \beta \text{ .} \qquad \text{(A-12)}$$

Note that equation (A-12) is similar to equation (A-4), except that now we have:

$$\beta = [1/A] [1/\sqrt{(2\pi)}] \int_{-\infty}^{U} e^{-\frac{1}{2}(y'-\sigma)^2} dy' \text{.} \qquad \text{(A-13)}$$

Note that now $0 < \beta < 1$, which means that the conditional expected value is always less than or equal to the unconditional expected value. In the case of very large outstanding balance on the mortgage, then ln(b) and U are also very large, and the values of A and β approach 1. In this case the conditional and unconditional expected values are nearly equal. In the limit, as b goes to ∞, they are equal.

Since the X, μ, and σ are actually time dependent, we rewrite equations (A-12) and (A-13) as:

$$E[X(t)|x < b(t)] = e^{\mu t+\frac{1}{2}\sigma^2 t} \beta \text{ ,} \qquad \text{(A-14)}$$

$$\beta = [1/A] [1/\sqrt{(2\pi)}] \int_{-\infty}^{U(t)} e^{-\frac{1}{2}(y'-\sigma\sqrt{t})^2} dy' \text{,} \qquad \text{(A-15)}$$

where

$$U(t) = \{\ln[b(t)] - \mu t\}/\sigma\sqrt{t} \text{ ,} \qquad \text{(A-16)}$$

and

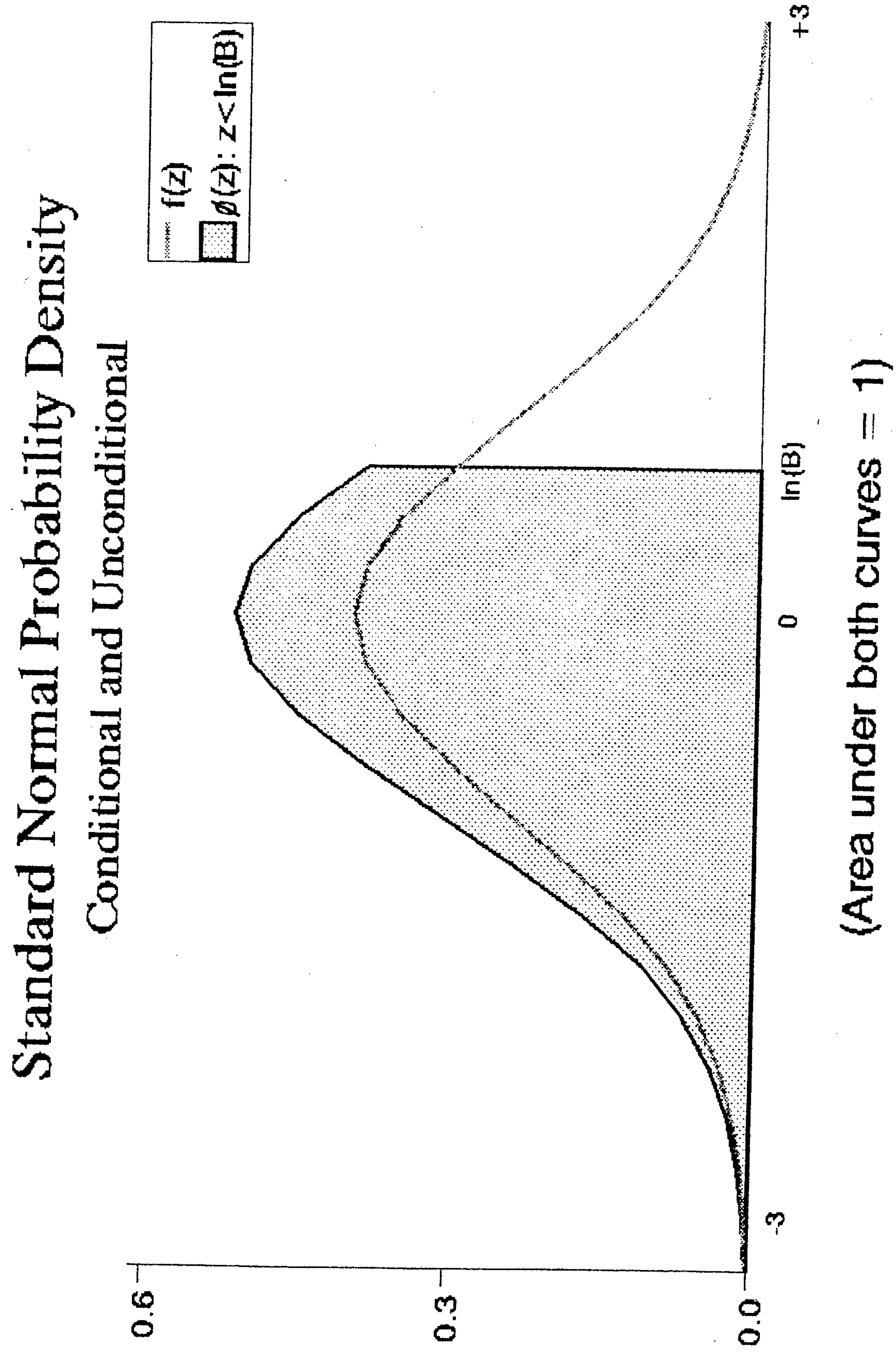
$$A = [1/\{\sigma\sqrt{t} \sqrt{(2\pi)}\}] \int_{-\infty}^{\ln[b(t)]} e^{-\frac{1}{2}[(y-\mu t)/\sigma\sqrt{t}]^2} dy. \qquad \text{(A-17)}$$

Equations (A-14) through (A-17) specify the desired conditional expected value. Note that equation (A-14) can also be written as:

$$E[H(t)|h < B(t)] = H_0 e^{\mu t+\frac{1}{2}\sigma^2 t} \beta \text{ .} \qquad \text{(A-18)}$$

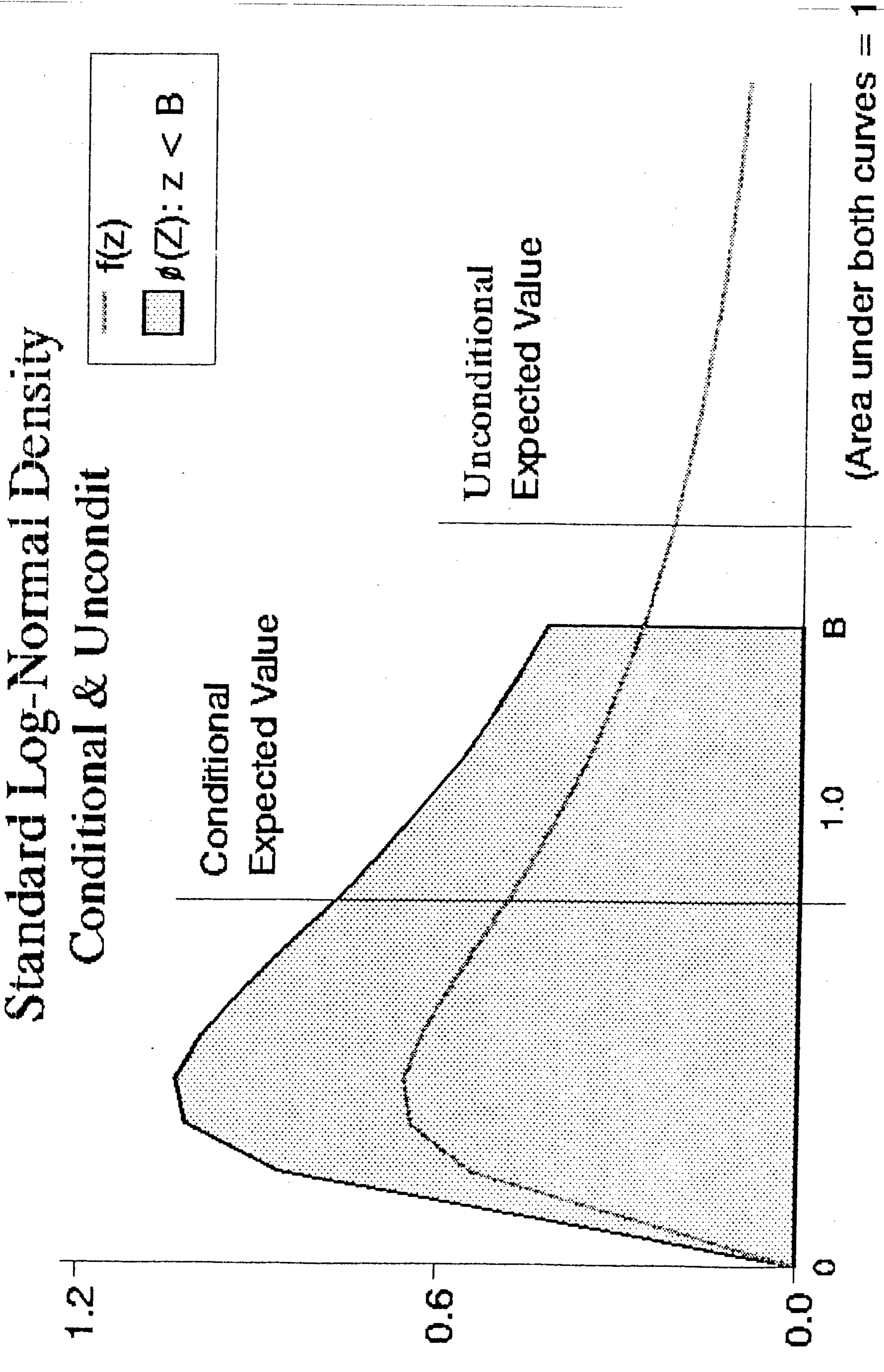
■ ■ ■

Figure A-1



Standard Normal Probability Density
Conditional and Unconditional
f(z)
ø(z): z<ln(B)
0.6
0.3
0.0
-3
0
ln(B)
+3
(Area under both curves = 1)

Figure A-2


Standard Log-Normal Density
Conditional & Uncondit
f(z)
ø(Z): z < B
Conditional
Expected Value
Unconditional
Expected Value
1.2
0.6
0.0
0
1.0
B
(Area under both curves = 1)

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