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Spatial Weight Matrices and Their Use As Baseline Values and Location-Adjustment Factors in Property Assessment Models

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Abstract

Property assessment models, for the purpose of mass appraisal and taxation, estimate the market price of real estate as a function of its location and physical characteristics. Locational effects, which affect multiple properties in an area, typically are established separately from property-specific effects. Baseline prices are established based on neighborhood or boundary demarcations, then a regression framework gives the adjustments to this baseline based on property-specific characteristics. As an alternative, baseline prices based on physical characteristics are first established, then location adjustments are applied as factors in the regression. This article shows how the spatial weight matrix in a spatial lag regression can be used as either a locational baseline value or as a location-adjustment factor, depending on the model specification.

Introduction

Several factors affect the value of real estate, with location known to be primary. Estimating locational effects on value, particularly in the area of mass appraisal, is done in a regression framework with proxy measures for location. Before the use of geographic information systems (GISs), locational effects were included in the model using variables that delineate neighborhoods, districts, or submarkets, either judged to be distinct or shown to be different using statistical techniques that analyze patterns of demography, crime, social trends, and other characteristics.

The introduction of GISs allowed for spatial relationships and distance-based variables to be included in the regression. One popular approach is to run the regression without locational effects and use GISs on the residuals to develop location-adjustment factors (Gallimore, Fletcher, and Carter, 1996; McCluskey et al., 2000). Surface-fitting techniques such as inverse distance weighted, spline, or kriging are used to construct a residual surface. Because residuals denote overprediction and underprediction, the GIS estimates of the surface are used to adjust for the overvaluation and undervaluation of a property within the area to estimate a location-influence variable. This variable is then included as a location-adjustment factor in the regression.

The problem with the residual approach is that it assumes the residuals capture no other omitted variables except for location. Thus, the previous approach can be understood more as a method to tighten the fit of the regression than as a latent estimate of location. Another approach, the location value response surface (LVRS) analysis, introduced by O'Connor (1982) and extended and applied to different markets by Eichenbaum (1989, 1985), Eckert, O'Connor, and Chamberlain (1993), O'Connor and Eichenbaum (1988), and Ward, Weaver, and German (1999), estimates the price surface, determines value-influence centers (VICs) using peaks and troughs or "hotspots," calculates distance to these centers, and regresses price on x-y coordinates and the distance to VICs. The ratio of the predicted price to the average estimated price is the local adjustment factor, with a mean of 1. In particular, better locations have a factor of more than 1 and the poorer locations have a factor of less than 1. This estimated location-adjustment variable is included in the hedonic regression of price.

Spatial correlation is taken into account in surface-fitting methods, because neighboring prices weighted by distance are used in the estimation. Spatial lag models, on the other hand, explicitly incorporate a measure of spatial correlation as an autoregressive term in the regression. Spatial econometric methods are developed, discussed, or reviewed in Anselin (1988), Anselin and Bera (1998), Basu and Thibodeau (1998), Dubin (1998), Dubin, Pace, and Thibodeau (1999), Kelejian and Prucha (1998, 1999), and Kelejian and Robinson (1993, 1995) and have been applied to different fields in the social sciences. Testing and correcting for spatial correlation in hedonic pricing models are widely used practices in econometric applications.

This article combines both techniques by illustrating how the spatial lag term can be normalized to work as a location-adjustment factor in the LVRS framework. The spatial term is similar to the LVRS analysis in that it is constructed from neighboring prices weighted by distance. Because the inclusion of spatial lags eliminates or reduces spatial correlation in the residuals, the spatial lag term has high explanatory power as a latent estimate for location. The article shows that using a normalized spatial lag variable significantly improves the fit of predicted prices in the additive-multiplicative framework of mass appraisal models.

Hedonic and Spatial Lag Models

In real estate economics, estimating property values from sales prices typically takes the form of hedonic regressions. Hedonic models assume that the item of interest, say property value, can be measured by decomposable characteristics, such as house size, lot square footage, and number of bedrooms. A hedonic regression treats these attributes separately and estimates the contributory value (in the case of an additive model) or elasticity (in the case of a log-linear model) of each attribute.

Estimating property value in terms of prices of sold parcels is specified by a general equation

$$price = \beta_0 \left(Z_1^{\beta_1} Z_2^{\beta_2} ... Z_3^{\beta_3}\right) \exp\left[\sum_{i=1}^{n} \gamma_i X_i + \sum_{j=1}^{m} \gamma_j D_j + \varepsilon\right] \qquad (1.1)$$

or in its natural log form as

$$lnprice = \ln \beta_0 + \beta_1 \ln Z_1 + ... + \beta_s \ln Z_s + \sum_{i=1}^{n} \gamma_i X_i + \sum_{j=1}^{m} \delta_j D_j + \varepsilon . \qquad (1.2)$$

The coefficients $\beta_1 ... \beta_s$ are elasticities and therefore the variables $Z_1 ... Z_s$ are continuous variables typically associated with size (house size, land size, and so on). The n coefficients γ measure the growth in price for a unit change in the variable X and are associated with other continuous variables such as age or distance. The m coefficients δ are adjustment factors based on the dummy variables D. The error ε is assumed to be $\left(0,\ \sigma_\varepsilon^2 I\right)$.

Spatial lag models can be interpreted as a specific form of equation 1.1 when errors fail the $iid\left(0,\ \sigma_\varepsilon^2 I\right)$ assumption and are instead spatially correlated. With spatial autocorrelation in the data, hedonic model estimation using ordinary least squares (OLS) regression of equation 1.2 generates inefficient estimates and therefore incorrect test statistics. Spatial autocorrelation occurs when prices in one location are correlated with prices in neighboring locations so that, for neighboring locations i and j, $E(price_i,\ price_j) \neq 0$ or alternatively $E(\varepsilon_i,\ \varepsilon_j) \neq 0$.

When the dependent variable exhibits spatial autocorrelation, a spatial lag term is constructed using a weighted average of the values in nearby locations and is added to equation 1.2,

$$lnprice = \ln \beta_0 + \alpha Wy + \beta_1 \ln Z_1 + ... + \beta_s \ln Z_s + \sum_{i=1}^{n} \gamma_i X_i + \sum_{j=1}^{m} \delta_j D_j + u, \qquad (2)$$

where α is the coefficient on the spatially lagged variable Wy, W is the weight matrix, and u is $iid\left(0,\ \sigma_\varepsilon^2 I\right)$.

Spatial Lag As Baseline Values

Consider estimating property values for single-family and multifamily homes in the borough of Queens, New York. For the purpose of assessment, single-family homes and multifamily homes with two and three residential units are considered in the same tax class, are assessed similarly, and are given the same tax rate. Consistent valuation among properties is important, because taxes are based on assessed values. Properties that have the same physical characteristics—for example, identical homes in terms of measurable physical characteristics such as square footage, number of bedrooms, land size, age, and so on—and are in similar locations—for example, are on the same block—must be given the same value. This requirement means that the spatial lag term *Wy*, which is a weighted average of neighboring prices, must be measured in the unit where consistency must be achieved. In this case, this level is assumed to be the block level, so that *Wy* is the weighted average of prices of neighboring blocks. The block level is the smallest unit required for consistency. Areas typically are groups of blocks, districts, or neighborhoods.

Thus, a parcel *k* in block *l* is described by rewriting equation 2 at the parcel level,

$$\begin{aligned} lnprice_{kl} = {} & \alpha W y_l \\ & + \left\{ \ln \beta_0 + \sum_{g=1}^{s} \beta_g \ln Z_{g_{kl}} + \sum_{i=1}^{n} \gamma_i X_{i_{kl}} + \sum_{j=1}^{m} \delta_j D_{j_{kl}} \right\} \\ & + u_{kl} \\ \Rightarrow {} & \textit{block} \text{ baseline price} + \textit{parcel} \text{ physical characteristics} + \text{error}. \end{aligned} \tag{3}$$

Using equation 3 means all parcels in the same block *l* have the same base price based on neighboring blocks and, for each parcel *k* within this block, the base price is adjusted for individual characteristics. If a block consists of identical homes (say, a row of brownstones), then the physical characteristics are identical and will adjust the baseline block price similarly. Thus, all identical homes in the same block will be priced the same.

We estimate equation 3 for Queens. Block *x* and *y* coordinates are used to compute the spatial lag term *Wy*. The dataset consists of 8,156 parcels sold between the first quarter of 2010 and the second quarter of 2012. Sales data are compiled by the Department of Finance, City of New York and are published on its website. Data were cleaned for non-arms-length transactions; in particular, they were cleared of foreclosure sales, sales in which one party was a public entity, sales in which one party was a financial institution, sales that indicated a transfer between relatives, and sales that transferred more than once within 1 year. The data were restricted to the borough of Queens because it has the greatest number of homes among the five boroughs of the City of New York.

Time-Trend Adjustment

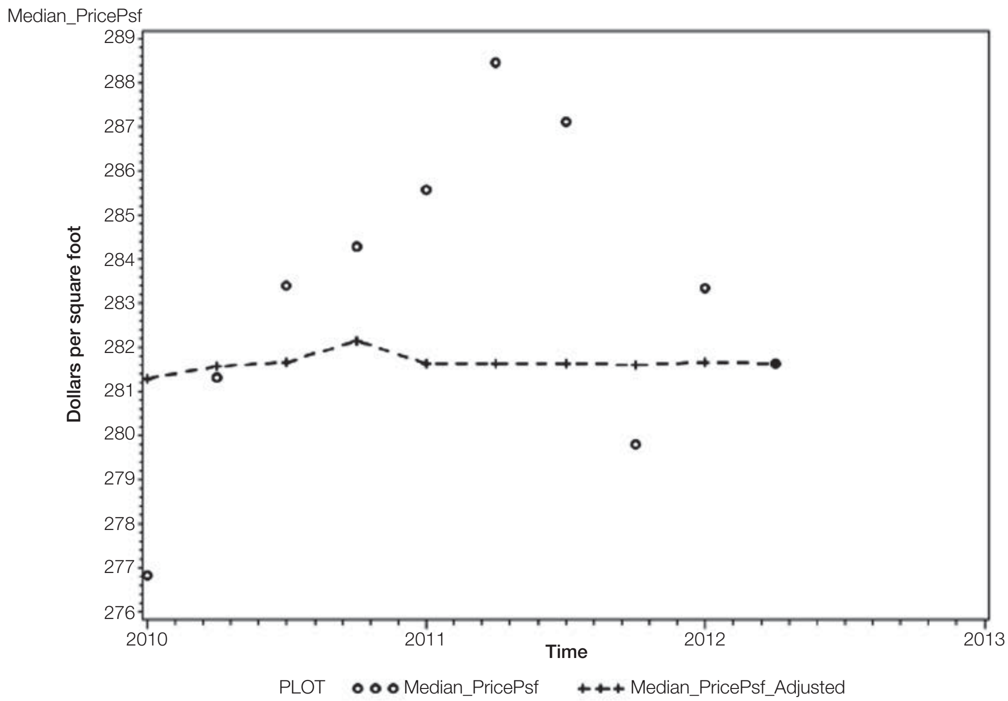
Because sales occurred during different periods, a trend must be accounted for before creating spatial lags. A time-trend regression of the log of price per square foot (lnpsf) on quarterly dummies was run to detrend the data to the end of period,

$$\ln psf = \alpha + \beta_i * \sum_{i=1}^{9} qt_i + \varepsilon, \tag{4}$$

where *qt* are the time dummies excluding the second quarter of 2012 as the base period. The median price per square foot (psf) and the time-adjusted median are plotted in exhibit 1. The dots are the actual median price psf across time, and the line shows that the time-adjusted median price psf is now stable or detrended around the base period's median of $281.64.

Exhibit 1

Median Price



Psf = per square foot.

Note: Prices were detrended to second quarter of 2012 level.

Spatial Lag Regression

The map in exhibit 2 displays the time-adjusted price psf and shows that a sufficient number of sales are spread out across the borough of Queens. A stepwise regression was run to determine the significant physical characteristics. The result of the stepwise regression in exhibit 3 is labeled OLS. The dependent variable is the log of detrended price psf (lnpricepsfadj), the independent variables are log of square footage of living area (lnsfla), log of land area (lnlnd_area), garage square footage (gar_sqft), age since alteration (altage), number of stories (stories), and dummy variables of whether the property has a basement (basement), whether it is a two-family or three-family home, its style (row, cape_cod, conventional, old_style), and exterior construction (alum_vinyl, composition, frame).

The adjusted R-square and the coefficient of dispersion (COD) are measures of fit for our regressions. The COD is used to measure uniformity in assessments. If we denote the error in the regression by the ratio of predicted to actual sales, then the COD measures the average departure or deviation of this calculated ratio at around 1. A high COD suggests a lack of equality and uniformity among individual assessments. For residential properties, the maximum allowable COD is 15 percent.

Exhibit 2

Spatial Distribution of Price per Square Foot in Queens, New York

The COD of the OLS regression is more than 15 percent and its adjusted R-square is low, at 45.68 percent. We did a test for spatial correlation on the OLS residuals using Moran's I. The value of Moran's I on the slope coefficient of the lagged residual was 0.596 with a t-statistic of 101, indicating the presence of spatial correlation.

To correct for spatial correlation, we added a spatial lag term based on neighboring blocks. The weight matrix was determined based on which spatial regression had the best COD and adjusted R-square. We did a k nearest neighbor estimate, starting with $k = 5$ to $k = 20$, and a minimum distance estimate using 2,354 feet as its threshold (about three New York City blocks). The results are in exhibit 3. Our final regression uses the minimum distance estimator because it has the best COD and R-square and its median ratio is closest to 1. The final regression model drops the insignificant variable (basement) and is given in exhibit 4, together with a fit plot of the predicted versus actual sales price.

Exhibit 3

Regression Models

dependent variable = lnpricepsfadj

	OLS		Spatial Regressions									
			k = 5		*k = 10*		*k = 15*		*k = 20*		*Min Dist*	*2,354 ft*
Adj R-Sq	45.68%		69.53%		70.64%		71.09%		71.45%		71.58%	
COD	19.81%		14.44%		14.13%		14.04%		13.94%		13.84%	
Median ratio	0.98872		0.98937		0.99214		0.99088		0.99101		0.99271	
Variable	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**
Intercept	9.0287	< .0001	4.71724	< .0001	4.56173	< .0001	4.44707	< .0001	4.37964	< .0001	4.32244	< .0001
lnsfla	– 0.69028	< .0001	– 0.56583	< .0001	– 0.5893	< .0001	– 0.60306	< .0001	– 0.61368	< .0001	– 0.64323	< .0001
lnlnd_area	0.22293	< .0001	0.12651	< .0001	0.11731	< .0001	0.11981	< .0001	0.12199	< .0001	0.12235	< .0001
gar_sqft	0.00041569	< .0001	0.00019895	< .0001	0.00017944	< .0001	0.00016363	< .0001	0.0001572	< .0001	0.00015271	< .0001
altage	– 0.00045772	0.0013	– 0.00029895	0.0051	– 0.00031581	0.0026	– 0.00028551	0.0061	– 0.00033386	0.0012	– 0.00041541	< .0001
stories	0.05046	< .0001	0.03518	< .0001	0.03673	< .0001	0.03844	< .0001	0.03686	< .0001	0.04059	< .0001
basement	0.07023	< .0001	0.0076	0.5325	0.00561	0.6389	0.01165	0.3256	0.00763	0.5172	0.00937	0.4252
two_family	0.01455	0.0274	0.07053	< .0001	0.07942	< .0001	0.08303	< .0001	0.08656	< .0001	0.08881	< .0001
three_family	0.05368	0.0001	0.11642	< .0001	0.12917	< .0001	0.13354	< .0001	0.13979	< .0001	0.14807	< .0001
frame	– 0.0773	< .0001	– 0.03732	< .0001	– 0.03736	< .0001	– 0.03818	< .0001	– 0.03711	< .0001	– 0.03403	< .0001
alum_vinyl	– 0.12532	< .0001	– 0.06046	< .0001	– 0.05317	< .0001	– 0.04911	< .0001	– 0.04727	< .0001	– 0.0466	< .0001
composition	– 0.10738	< .0001	– 0.03847	< .0001	– 0.03101	< .0001	– 0.02742	< .0001	– 0.02511	< .0001	– 0.02171	0.0007
row	– 0.09653	< .0001	– 0.0569	< .0001	– 0.06255	< .0001	– 0.05998	< .0001	– 0.05679	< .0001	– 0.04918	< .0001
cape_cod	– 0.06278	< .0001	– 0.06217	< .0001	– 0.06295	< .0001	– 0.06236	< .0001	– 0.06393	< .0001	– 0.05639	< .0001
conventional	– 0.09186	< .0001	– 0.04308	< .0001	– 0.04354	< .0001	– 0.04169	< .0001	– 0.04182	< .0001	– 0.03924	0.0002
old_style	– 0.11756	< .0001	– 0.05392	< .0001	– 0.05353	< .0001	– 0.05173	< .0001	– 0.04846	< .0001	– 0.04035	< .0001
ln_spatial_lag			0.72698	< .0001	0.79647	<.00	0.82811	< .0001	0.85175	< .0001	0.89742	< .0001

Adj R-Sq = adjusted R-square. COD = coefficient of dispersion. OLS = ordinary least squares.

Exhibit 4

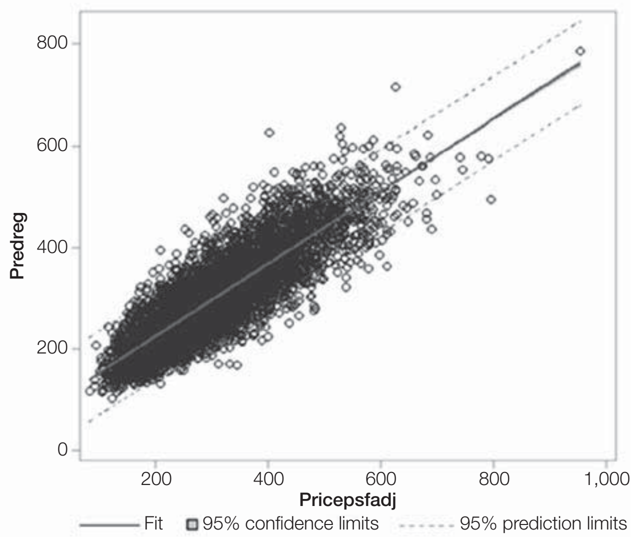
Final Regression Model and Predicted Regression (predreg) Versus Actual Sale Price

Spatial Regression

dependent variable = lnpricepsfadj

Adj R-Sq	71.59%
COD	13.84%
Median ratio	0.9924

Variable	Parameter Estimate	Pr > |t|
Intercept	4.32723	< .0001
lnsfla	− 0.64291	< .0001
lnlnd_area	0.12232	< .0001
gar_sqft	0.0001512	< .0001
altage	− 0.0004109	< .0001
stories	0.03991	< .0001
two_family	0.0887	< .0001
three_family	0.14735	< .0001
frame	− 0.03375	< .0001
alum_vinyl	− 0.04664	< .0001
composition	− 0.02167	0.0007
row	− 0.04892	< .0001
cape_cod	− 0.05616	< .0001
conventional	− 0.03925	0.0002
old_style	− 0.03973	0.0001
ln_spatial_lag	0.89792	< .0001



Observations	8,156
Parameters	2
Error DF	8,154
MSE	1,774.4
R-square	0.7308
Adj R-square	0.7308

Adj R-Sq = adjusted R-square. COD = coefficient of dispersion. DF = degrees of freedom. MSE = mean square error.

Spatial Lag As Location Adjustments

An alternative and commonly used approach to property valuation is using an additive model to establish a base price based on contributory value, then using the multiplicative model (log-linear model) to apply adjustments to the estimated base price. Using our final model in the previous section, the first regression would be specified as

$$price = \beta_1 * landarea + \beta_2 * sfla + \upsilon, \quad (5)$$

where *sfla* is the square feet of living area and the dependent variable is time-adjusted price. Note that the regression is in price level, not in price psf, so that the coefficient β measures the dollar contribution of an additional square foot of land or building. The intercept is also suppressed to ensure positive prices. Properties that are identical in size and land area have the same base price regardless of location, exterior finish, age, and so on.

Adjustments to the base price for location, age, and finish are estimated in a log-linear framework. Let $\widehat{price}$ denote the estimated price in equation 5, then adjustments to this base price are modeled as in equation 1.1,

$$price = \beta_0 \left(\widehat{price}\right) \exp\left[\sum_{i=1}^{n} \gamma_i X_i + \sum_{j=1}^{m} \gamma_j D_j + \varepsilon\right], \quad (6.1)$$

or alternatively as in equation 1.2 in log form,

$$lnPrice = \ln \beta_0 + \beta_1 \ln \hat{price} + \sum_{i=1}^{n} \gamma_i X_i + \sum_{j=1}^{m} \delta_j D_j . \quad (6.2)$$

The spatial lag estimated in the previous section can be used as a location-adjustment factor in equation 6.2. Take the median of the spatial lag term for the whole borough and use it as a normalizing factor. For parcel *k* in block *l*, define its location adjustment as

$$locadj_{kl} = \frac{Wy_1}{median(Wy_1, \ldots_i Wy_n)}, \quad (7)$$

where *n* is the total number of blocks. Then $locadj_{kl}$ is a ratio that is applied to increase or decrease the base price $\hat{price}$. Values greater than 1 indicate locations that are priced higher than the borough median and, similarly, values that are less than 1 are in sale areas with values lower than the borough median.

Exhibit 5 contains the regression specification (equation 6.2) using the variables in the previous section. The dependent variable is the log of adjusted price (lnpricepsfadj); the independent variables are log of the estimated base price (ln_basePrice), log of square footage of living area (lnsfla), log of land area (lnlnd_area), garage square footage (gar_sqft), age since alteration (altage), number

Exhibit 5

Location-Adjustment Model

dependent variable = lnpricepsfadj

	OLS		Spatial/Location Adjustment	
Adj R-Sq	40.42%		68.68%	
COD	19.83%		13.89%	
Median ratio	0.98898		0.99159	
Variable	**Parameter Estimate**	**Pr > |t|**	**Parameter Estimate**	**Pr > |t|**
Intercept	6.35184	< .0001	6.62847	< .0001
ln_basePrice	0.52326	< .0001	0.48516	< .0001
gar_sqft	0.00041075	< .0001	0.00016903	< .0001
altage	– 0.0004085	0.0042	– 0.00040783	< .0001
stories	0.04113	< .0001	0.04733	< .0001
two_family	0.00694	0.2646	0.10106	< .0001
three_family	0.01786	0.1775	0.15487	< .0001
frame	– 0.06867	< .0001	– 0.04571	< .0001
alum_vinyl	– 0.12631	< .0001	– 0.04532	< .0001
composition	– 0.10805	< .0001	– 0.01771	0.0058
row	– 0.09928	< .0001	– 0.01946	0.0845
cape_cod	– 0.06271	< .0001	– 0.07669	< .0001
conventional	– 0.08499	< .0001	– 0.02028	0.0558
old_style	– 0.1047	< .0001	– 0.0226	0.0292
ln_locadj			0.88468	< .0001

Adj R-Sq = adjusted R-square. COD = coefficient of dispersion. OLS = ordinary least squares.

of stories (stories), and dummy variables of whether it is a two-family or three-family home, its style (row, cape_cod, conventional, old_style), and exterior construction (alum_vinyl, composition, frame). The location adjustment is the variable *ln_locadj*. The addition of the location-adjustment factor significantly increases the adjusted R-square to 68.68 percent and decreases the COD to 13.89 percent.

Summary

This article showed how spatial lags can be incorporated into property assessment models. We presented two methods in which spatial lags can be used: first, as a base price established by location (blocks, districts, neighborhoods) and, second, as a location-adjustment factor. The choice of which specification to use depends on the objective and use of the models. For the case in which total value is of interest, such as in assigning estimated prices to unsold properties or microstudies and macrostudies of market trends, then the spatial lag model is sufficient. For cases in which contributory values are required, such as in valuing new properties or estimating the addition of square footage of buildings in progress, then the second approach is applicable. In each case, the use of spatial lags improves the model fit significantly.

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